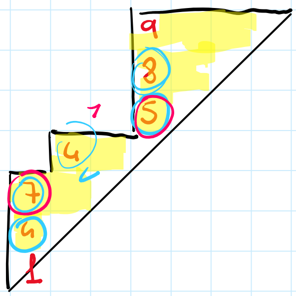
IPAC 2026 - Lecture 1

Tuesday, 2 June 2026 18:00

Conj: 2003-4

Proof: 2015

$$\nabla e_n = \sum_{\pi \in LD(n)} q^{dinv(\pi)} t^{area(\pi)} x^{\pi}$$



area = 11

dinv = 4

$x^{\pi} = x_1 x_4^2 x_5 x_7 x_8 x_9$

Thm: D'Adderio - Mellit
BHMPS
Gillespie-Gorsky-Griffin

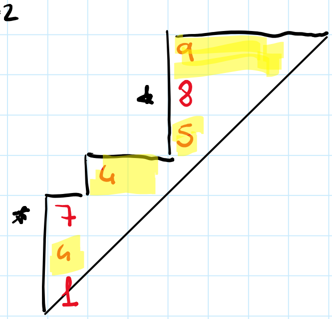
Conj: 2015

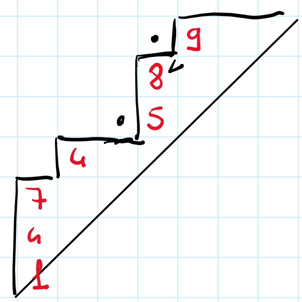
$$\Delta'_{e_{n-k-1}} e_n = \sum_{\pi \in LD(n)^{*k}} q^{dinv(\pi)} t^{area(\pi)} x^{\pi} = \sum_{\pi \in LD(n)^{\cdot k}} q^{dinv(\pi)} t^{area(\pi)} x^{\pi}$$

RISE VERSION

VALLEY VERSION

$k=2$

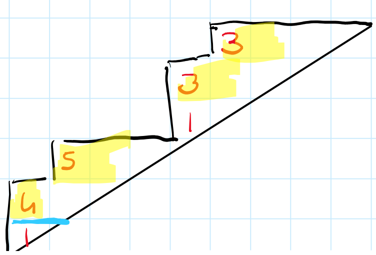




OPEN

$$e_{m,n} = \sum_{\pi \in LD(m,n)} q^{dinv(\pi)} t^{area(\pi)} x^{\pi}$$

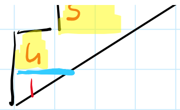
$n=6$



area = 7

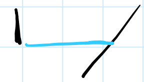
w_i = label in row i

a_i = horizontal dist.



$m = 9$

w_i = label in row i

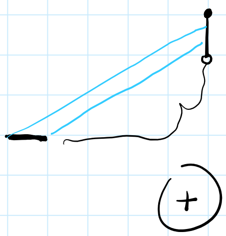
a_i = horizontal dist. 

it can be a fraction!

$$\mathrm{tdinv}(\pi) = \# \{ (i,j) \mid (a_i, i) \leq_{lex} (a_j, j) \leq_{lex} (a_i+1, i) \text{ and } w_i < w_j \}$$

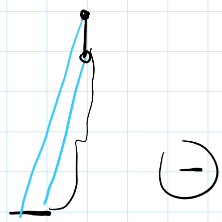
$i \to j$

i attacks j



$+$

$m > n$



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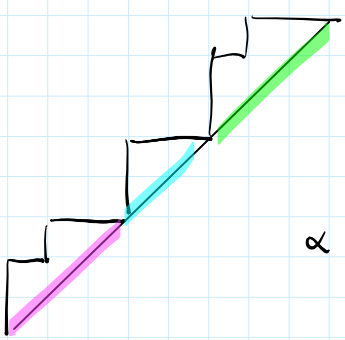
$m < n$

Theta operators $\quad f, g \in \Lambda \quad M = (1-q)(1-t)$

$$\Theta_f\, g = \Pi\, f[X/M]\, \Pi^{-1} g \qquad \Pi = \sum (-1)^i \Delta_{e_i}$$



$\alpha = 3, 2, 3$

∇C_α

$$\sum_{\alpha \models n} C_\alpha = e_n$$

D'Adderio - Mellit

2021 (Comp. (rise) Delta thm.

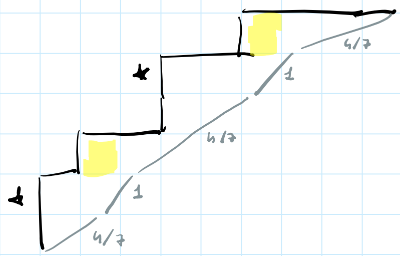
$$\Delta'_{e_{n-\dots}} e_n = \Theta_{e_k} \nabla e_{n-k}$$

$$\Delta'_{e_{n-k-1}} e_n = \Theta_{e_k} \nabla e_{n-k}$$

2021 Comp. (rise) Delta thm.

$$\Theta_{e_k} e_{m,n} \overset{?}{=} \sum_{\pi \in LD(m+k,\, n+k)^{*k}} q^{\mathrm{dinv}(\pi)} t^{\mathrm{area}(\pi)} x^{\pi}$$

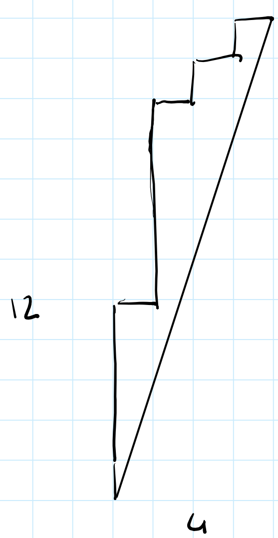
$q = 1$



$(4+2) \times (7+2)$

Gillespie - Gorsky - Griffin

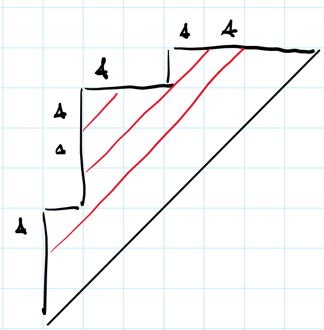
$$s^{\perp}_{(n-1)^{k-1}} e_{n,kn} = \Delta'_{e_{n-k-1}} e_{n+k} \quad \left(= \Theta_{e_k} \nabla e_n \right)$$



skewing

$n = 4$

$k = 3$

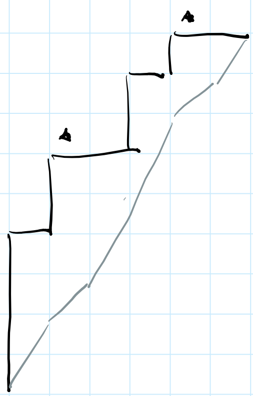


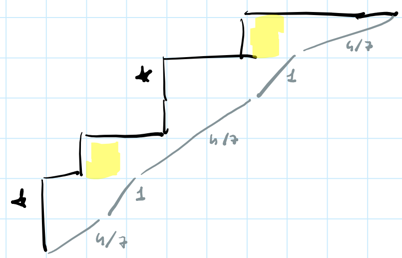
$$s^{\perp}_{(m-1)^k}\, e_{m,\,n+km} \stackrel{?}{=} \Theta_{e_k}\, e_{m,n}$$

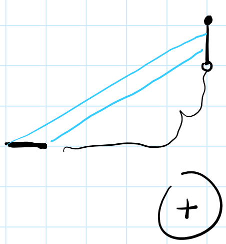
NOPE !!

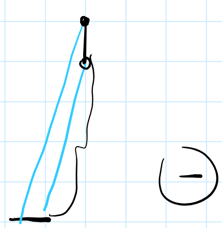
$$\langle s^{\perp}_{(m-1)^k}\, e_{m,\,n+km},\ h_d\, e_{n+k-d} \rangle \stackrel{\text{conj}}{=} \langle \Theta_{e_k}\, e_{n,m},\ h_d\, e_{m+k-d} \rangle$$

$d=0$

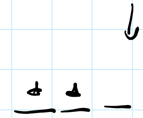






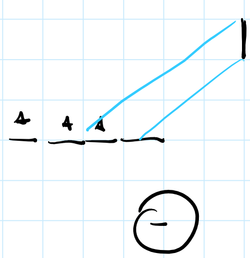


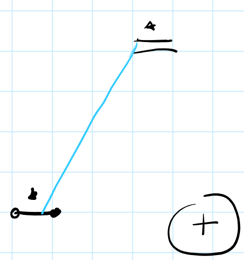
one big step

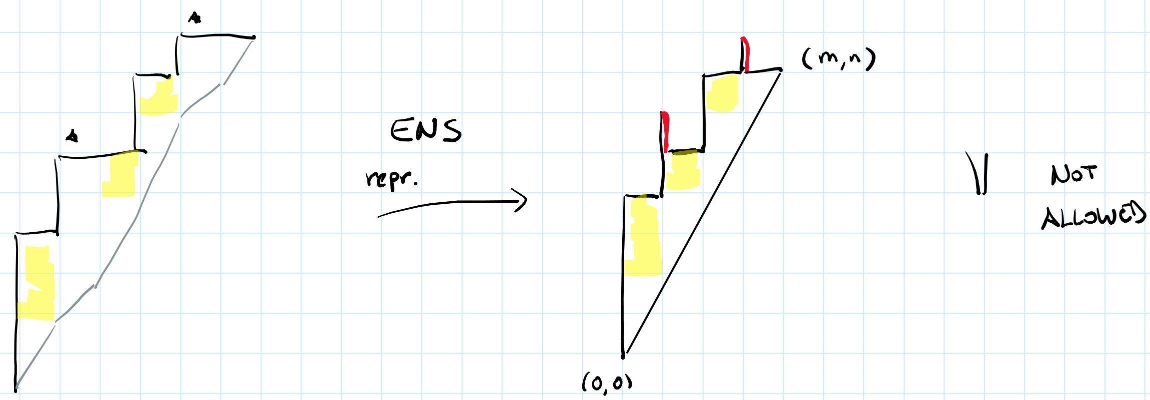


$m > n$

$m < n$






ENS
repr.
(m,n)
(0,0)
NOT
ALLOWED