**Take-Home Final Exam - Math 5800 - Fall 2025**

**Instructions:** Do Problems $1, 2, 3, 4$ below, and in addition do problem 133 from Chapter 1 and problems 25 and 33 from Chapter 2 of Stanley's *Enumerative Combinatorics, Volume I*, 2nd edition. For the problems from Stanley, you are free to consult his solutions, though if you do, you must fill in all missing details. Each problem is worth ten points.

**1**. Do exercise 2.11 from the "Notes on Rook Polynomials" from the course webpage.

**2**. Do exercise 2.12 from the "Notes on Rook Polynomials" from the course webpage.

**3**. Use the RSK algorithm (as described in class and also in Sagan's book, Chapter 7 of Stanley Volume 2, or Chapter 1 of my $q, t$-Catalan book) to show that for any positive integer $n$,

$$\left(\sum_{i\geq 1} x_i\right)^n = \sum_{\lambda\vdash n} f^\lambda s_\lambda(X),$$

where $X = \{x_1, x_2, \ldots\}$, $f^\lambda$ is the number of SYT of shape $\lambda$, and the sum on the right is over all partitions $\lambda$ of $n$.

**4**. For the labelled poset whose Hasse diagram is given below, compute the Möbius function $\mu[1,(m+1)k+2]$. (See Section 3.7 from Chapter 3 of Stanley's *Enumerative Combinatorics, Volume I*, 2nd edition for a recursive definition of the Möbius function of a poset.) Here $m,k$ are arbitrary positive integers. The poset consists of $m+1$ rows of vertices, with each row consisting of $k$ vertices, together with a minimal element labelled 1 and a maximal element labelled $(m+1)k+2$. within a given row the vertices are labelled in increasing order left-to-right. Each of the vertices $v$ in one of the bottom $m$ rows is connected to the three vertices in the row above which are immediately above, to the left, and to the right of $v$ (if they exist). In addition each of the vertices in the bottom row is connected to the minimal element 1, and each in the top row is connected to the maximal element $(m+1)k+2$.