Take Home Final Exam, Math 3610 Fall 2025

1. (10 pts.) Exercise 4, p. 411 from Section 7.5.

2. (10 pts.) Find the minimum value of

$$F(x, y, z) = x + y + 2z$$

subject to the constraint

$$x^2 + y^2 + z^2 \leq 1.$$

3. (10 pts.) Find the maximum and minimum values of

$$x^2 + 2y$$

subject to the constraints

$$2x - 3y = 5$$

and

$$x^2 + y^2 = 4.$$

4. (10 pts.) A container in the shape of a right circular cylinder with no top is to be constructed from 10 sq. ft of sheet metal. Find the dimensions (i.e. the radius r of the base and the height h) of the cylinder that maximize the volume.

5. (10 pts.) Evaluate the integral, or show it doesn't exist.

$$\int_{x^2+y^2+z^2\leq 5} \frac{1}{x^2 + y^2 + z^2}\, dx\, dy\, dz.$$

6. (10 pts.) Evaluate the integral.

$$\int_A x\, dx\, dy$$

where $A = \{(x, y) : 0 < x < \sqrt{\pi}\,,\ 0 < y < \sin x^2\}$.

7. (10 pts.) Evaluate the integral.

$$\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} (x^3 + 3y^2x)\, dy\, dx.$$

8. (10 pts.) Consider the integral

$$I = \int_T (x - 2y)\, dy\, dx$$

where T is the parallelogram with vertices $(2, 4)$, $(8, 6)$, $(12, 10)$ and $(6, 8)$. Use the transformation

$$u = x - y, \quad v = x - 3y$$

to evaluate I.