Solution to Math 104, Quiz 7

Given $y = \frac{1}{2}(e^x + e^{-x})$ with $x \in [0, 1]$ and rotate it around y-axis, what is the area of the surface? Recall the correct formula we should use is:

$$A = \int 2\pi x \sqrt{1 + f'^2} \mathrm{d}x \tag{1}$$

So we calculate:

$$f' = \frac{1}{2}(e^x - e^{-x}) \tag{2}$$

$$1 + f'^2 = 1 + \frac{1}{4}(e^{2x} - 2 + e^{-2x}) = \frac{1}{4}(e^{2x} + 2 + e^{-2x}) = (\frac{e^x + e^{-x}}{2})^2 \tag{3}$$

Note here we complete the term into a square as usual, finally we obtain:

$$A = \int_0^1 2\pi x \frac{e^x + e^{-x}}{2} \mathrm{d}x = \pi \int_0^1 xe^x + xe^{-x} \mathrm{d}x \tag{4}$$

The last step is an easy application of integration by parts:

$$\frac{1}{\pi} A = xe^x \Big|_0^1 - \int_0^1 e^x \mathrm{d}x - xe^{-x} \Big|_0^1 + \int_0^1 e^{-x} \mathrm{d}x = e - (e - 1) - e^{-1} + (-e^{-1} + 1) = 2(1 - \frac{1}{e}) \tag{5}$$

$$A = 2\pi(2 - \frac{1}{e}) \tag{6}$$

I provided the results for each of the four terms, you should do it by hand to check it