Math 104, Quiz 2

the problem is to solve the following differential equation using integral factor:

$$1 + \frac{2xdy}{ydx} = \frac{1}{y^2} \tag{1}$$

after multiply by y^2,we get

$$y^2 + \frac{2xydy}{dx} = 1 \tag{2}$$

Now the upshot is the left hand side of our equation is exactly $\frac{d(xy^2)}{dx}$, the computation is as follows, view y as a function of x,by **chain rule**, we should have:

$$\frac{d(xy^2)}{dx} = y^2\frac{dx}{dx} + 2xy\frac{dy}{dx} = y^2 + 2xy\frac{dy}{dx} \tag{3}$$

so our original equation can be now write as:

$$d(xy^2) = dx \tag{4}$$

so the solution is of the form:

$$xy^2 = x + c \tag{5}$$

substitute the initial condition which is $y(1) = 2$ we get $4 = 1 + c$, so $c = 3$ then finally taken into consideration that $x > 0, y > 0$, we have

$$y = \sqrt{1 + \frac{3}{x}} \tag{6}$$