Solution to homework 7

1. No need to explain this one, too easy and everyone got the right answer.

2. $a_n = \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$ multiply both the numerator and denominator by $\sqrt{n+1}+\sqrt{n}$ we get:
   $a_n = \frac{1}{(\sqrt{n+1}+\sqrt{n})^2}$,the denominator diverge to $+\infty$ which imply our sequence converges to 0.

3. $a_n = (n+e^n)^{\frac{1}{n}}$,construct $b_n = (e^n)^{\frac{1}{n}} = e$ as the constant sequnce, and form the ratio of a_n, b_n and take limit: $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \lim\limits_{n\to\infty} (\frac{n+e^n}{e^n})^{\frac{1}{n}} = \lim\limits_{n\to\infty} (1+\frac{n}{e^n})^{\frac{1}{n}} = 1^0 = 1$

4. If you didn'f use the following method which I pointed out in some of your homeworks, please take a look.
   The problem asks to find the Maclaurin series of the function $f(x) = \frac{x^2}{1+x^3}$, the correct way to do it is NOT use the general formula about Taylor series, which is too complicated since you need to calculate all the higher derivatives.
   Instead, we should notice first: $\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + ...$ is the Maclaurin series for $\frac{1}{1+x}$. Therefore simply substitude x by x^3 we obtatin $\frac{1}{1+x^3} = 1 - x^3 + x^6 - x^9 + x^{12} - x^{15} + ...$ is the corresponding Maclaurin series for $\frac{1}{1+x^3}$. Finally recall the Maclaurin series for the product of two function is the product of their own Maclaurin series. So our function $f(x) = x^2 * \frac{1}{1+x^3}$ has Maclaurin series $x^2 - x^5 + x^8 - x^{11} + x^{14} - x^{17} + ...$

5. Twice application of integration by parts, if you didn't get 2 as the answer, find a large enough sheet of paper and write neatly every step, if you still didn't get the correct answer, come to me.

6. $a_n = n(2^{\frac{1}{n}} - 1)$, we can write it as $a_n = \frac{(2^{\frac{1}{n}}-1)}{\frac{1}{n}}$, set $f(x) = \frac{2^x-1}{x}$ it suffices to calculate of $\lim\limits_{x\to 0+} f(x)$, this is $\frac{0}{0}$-type limit, therefore we apply L'Hopital's Rule even though he bought this rule rather than invented this rule. Recall $(2^x)' = (\log 2)2^x$ and substitute back, we get the limit is $\log 2$

7. I said polynomials are much slower than geometric growth, $a_n = \frac{(-2)^n}{n}$, therefore by n-th term test if you want (ratio,root test also work here with limit -2 and 2 respectively), the sequence diverges.

8. We talked about this during recitation, turn the sequence upside down and use $e = \lim\limits_{n\to\infty} (1+\frac{1}{n})^n$ we conclude the limit is $\frac{1}{e}$