What you should know from Homework 2

1. No much to say about problem 1, just keep in mind the relateion:

$$x \approx sin(x) \approx tan(x) \approx sin^{-1}(x) \approx tan^{-1}(x) \tag{1}$$

so it is easy to check the answer is $\frac{1}{2}$, you certainly can use L'Hopital Rule for this problem.

2. Chain Rule is always vital, so be extremely careful whenever you are working with chain rule. This problem is easy:

$$f(x) = e^{-x^2}, f'(x) = e^{-x^2}(-2x), f'(1) = e^{-1}(-2) = -2e^{-1} \tag{2}$$

3. This is the type of problem that is for free, very easy, just substitute the given function and calculate

4. You certainly can try them one by one, however solving it directly might be faster:

$$\frac{dy}{dx} = 4xy, \Rightarrow \frac{dy}{y} = 4xdx \tag{3}$$

so we integrate on both side:

$$\int \frac{dy}{y} = \int 4xdx, \Rightarrow \ln y = 2x^2 + C, \Rightarrow y = e^{2x^2+C} \tag{4}$$

Now put $C = 0$ we get the option (C)

5. This problem is important, remember the funtion is decreasing if the derivative is negative, and increasing if it is positive. So it amounts to find out the interval where $f(x) = \frac{\ln x}{x}$ satisfy $f'(x) < 0$, use Quotient Rule,

$$f'(x) = \frac{x\frac{1}{x} - 1\ln x}{x^2} = \frac{1 - \ln x}{x^2} \tag{5}$$

so we need to solve $1 - \ln x < 0$ which is just $x > e$

6. This one has a typo, so I will solve the case for $t = 10$ and simply write down the answer for $t = 100$. here is the solution anyway:

$$\frac{dP}{dt} = \frac{P}{20}(1 - \frac{P}{4000}), P(0) = 1000 \tag{6}$$

$$dt = \frac{dP}{\frac{P}{20}(1 - \frac{P}{4000})} = (\frac{A}{P} + \frac{B}{4000 - P})dP \tag{7}$$

compare the coeffiecients (we will learn more about partial fraction, which has the argument here as a prototype), we get $A = B = 20$ so we integrate on both side of the equation and get:

$$\int \frac{dt}{20} = \int \frac{dP}{P} + \int \frac{dP}{4000 - P} \tag{8}$$

$$\frac{t}{20} + C = \ln P - \ln(4000 - P) = \ln \frac{P}{4000 - P} \tag{9}$$

Now do some linear algebra, put $t = 0, P = 1000$, we get $C = -\ln 3$, next, move the terms in a smart way,put $t = 10$ you should get the following (which is actually easy):

$$e^{\frac{1}{2}} = \frac{3P}{4000 - P} \tag{10}$$

finally,do a little bit more algebra, it gives :

$$P = \frac{4000}{1 + 3e^{-\frac{1}{2}}}, P = \frac{4000}{1 + 3e^{-5}} \text{(for those who did t=100)} \tag{11}$$

In summary, this example is long and hard to compute sometimes, but the whole point is just **partial fractions integral**

7. Again you can check the options one by one, however that is not a good nor a fast way. After seperating the variables, and integrate,you should get:

$$\int y^2 dy = \int \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx \tag{12}$$

use substitution, set $x = u^2, u = \sqrt{x}$, we get:

$$\frac{y^3}{3} = -2e^{-\sqrt{x}} + C, y(0) = 3 \Rightarrow 9 = -2 + C \Rightarrow C = 11 \tag{13}$$

If you have trouble with this problem, either you didn't seperate the variables in a right way or you didn't realize you should use substitution to intgrate the right hand side of equation (12).

8. This is similar to the above one, after seperate the variables we need to integrate the following:

$$\int y dy = \int \frac{\ln x}{x} dx \tag{14}$$

use substitution $u = \ln x$, the answer is:

$$y^2 = \ln x^2 + C, y(1) = 2 \Rightarrow C = 4 \tag{15}$$

hundreds of people take the square root after this step as follows:

$$y = \sqrt{(\ln x)^2 + 4} = \ln x + 2 \tag{16}$$

I would say, **our life is easy** if the last equality hold. Remember you **can not** take square root nor take square opeartion seperately.

9. Notice this is linear after moving the x to the left and write $2tx - x = (2t - 1)x$, then apply the general formula. A quicker but equivalent way is to seperate the varibles, the computation is easy so is omitted.

10. seperate the variables, this problem is for free.