

## What you should know from Homework One

### 1

The first problem have essentially one difficulty, that is finding the Taylor series for  $e^{-x^2}$

You must be familiar with the Taylor series for  $e^x$  even if you are allowed a cheat sheet. Now the Taylor series for  $e^{-x^2}$  is just replacing  $x$  by  $-x^2$  as

$$e^x = 1 + x + \frac{x^2}{2} + \dots + \frac{x^5}{120} + \dots \quad (1)$$

$$e^{-x^2} = 1 + (-x^2) + \frac{x^4}{2} + \dots + \frac{-x^{10}}{120} + \dots \quad (2)$$

Note the problem asks the coefficient of the 10-th term, which is the coefficient of  $x^{10}$ , so should be  $-\frac{1}{120}$

You should always keep in mind if a term does not show up, it means it has coefficient 0.

The terminology is standard in math, we do talk about the 0-th term which is the constant term and write it as  $a_0$  in general. So the 10-th term here is actually the 11-th term in real word, so try to make sure you know what the question is asking for.

### 2

This one is easier than most of you expected, you need to compute the  $a_0, a_1, a_2$ , well the  $x^2$  term is easy, so let's focus on  $\arcsin(x)$ .

Note this is an odd function(why?), so its even Taylor coefficients are all 0(why?), therefore it remains to find  $a_1$  which equals the derivative of  $\arcsin(x)$  at  $x = 0$ , by the given hint, this is 1, therefore we conclude  $a_0 = 0, a_1 = 1, a_2 = 1$ (remember the  $a_2$  comes from  $x^2$  in our formula). Now sum up their square, we get 2.

### 3

Try to understand how I solved problem 1, then remember you also need to be familiar with the following:

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad (3)$$

Finally, I didn't take a lot of points or even took off points for minor errors, for example the sign for the coefficients, so maybe you get 10 points but that does not mean you did perfectly for everything. But if you are comfortable with all what I have said above, then you are good enough. The grading scheme for this homework is as follows:

5 points to start as long as you handed in and tried to solve these problems  
1/2/2 for the 3 problems depending on your solution.