Local-Global Principles and Their Obstructions

October 1–3, 2015, University of Pennsylvania

Talks are in DRL A6 unless stated otherwise.

Thursday

9:30-10:45 Registration and Tea/Coffee in the Lounge in DRL

10:45-11:45 David Harbater: Arithmetic curves and patches

12:00-1:45 Lunch break

– Talks on Thursday afternoon are in Lerner Center 101 –

1:45-2:45 Julia Hartmann: Algebraic groups, factorization, and Galois cohomology

3:00-4:00 Tea/Coffee in the Lounge in DRL

4:15-5:15 V. Suresh: Local-global principles with respect to discrete valuations of function fields of curves over complete discrete valued fields

Friday

–Talks on Friday morning are in Goddard Lab 101 –

9:30-10:30 R. Parimala: Reciprocity obstructions to Hasse principle over function fields of complete discrete valued fields

10:45-11:45 David Harbater: Comparison and descriptions of Tate-Shafarevich obstructions

12:00-1:45 Lunch break

1:45-2:45 Daniel Krashen: Local-global principles for Galois cohomology and cohomological invariants

2:45-3:15 Tea/Coffee in the Lounge in DRL

3:30-4:30 Asher Auel: Local-global obstructions for isotropy of quadratic forms over function fields

4:45-5:30 Exercise Session

6:30 Conference dinner at Penang (117 N 10th St)

Saturday

9:30-10:30 Yong Hu: A cohomological Hasse principle for fraction fields of two-dimensional local rings

10:45-11:45 Diego Izquierdo: Arithmetic duality theorems for function fields of curves over local fields and arithmetical applications

12:00-1:00 Daniel Krashen: Omissions, summary, open problems

1:00- Lunch (Sandwiches) and informal discussions in DRL A6

Rooms

- Math department Lounge, DRL 4th floor, 209 South 33rd Street
- DRL A6, 209 South 33rd Street
- Lerner Center 101 (Music Building), 201 South 34th Street
- Goddard Laboratories 101, 3710 Hamilton Walk

Abstracts for selected talks

Arithmetic curves and patching (David Harbater)

The talk will begin with an introduction to the topic of the workshop, including a discussion of local-global principles in the classical setting and how that motivates the material we will present. There will then be a discussion of curves over complete discrete valuation rings from algebraic and geometric points of view, including models of function fields, special and generic fibers, and reduction graphs. The notion of patches will then be presented.

Algebraic groups, factorization, and Galois cohomology (Julia Hartmann)

The talk will give a quick introduction to concepts central to the workshop theme, like linear algebraic groups, homogeneous spaces, torsors, Galois cohomology, and rationality properties. It will then use patches to construct a Mayer-Vietoris type sequence for the Galois cohomology of arbitrary linear algebraic groups, and a factorization result for rational connected linear algebraic groups.

Local-global principles with respect to discrete valuations of function fields of curves over complete discrete valued fields (Suresh)

Let K be a complete discretely valued field and F the function field of a curve over K. Let Ω_F be the set of discrete valuations of F. Let G be a connected linear algebraic group over F and X a projective or principal homogeneous space of G over F. We explain the validity of Hasse principle for X with reference Ω_F for certain classes of connected linear algebraic groups G defined over F. Applications to quadratic forms and central simple algebras will be given.

Reciprocity obstructions to Hasse principle over function fields of complete discrete valued fields (Parimala)

In this lecture we explain a reciprocity obstruction to Hasse principle for for varieties over function fields of curves over complete discrete valued fields, similar to the Brauer Manin obstruction in the case of number fields. Using this obstruction, we shall explain the construction of a principal homogeneous space under a nonrational torus over $\mathbb{C}((t))(E)$ for a suitable elliptic curve E over $\mathbb{C}((t))$ for which Hasse principle fails.

Comparison and descriptions of Tate-Shafarevich obstructions (David Harbater)

The obstruction to a local-global principle can be viewed as a generalization of the classical Tate-Shafarevich group for elliptic curves over number fields. For curves over complete discretely valued fields, there are several ways to formulate this obstruction, and the resulting objects can be compared. Questions include when they agree, when they are finite, when they are trivial, and how to give an explicit description of the obstructions. The current state of knowledge will be discussed.

Local-global obstructions for isotropy of quadratic forms over function fields (Asher Auel)

After a brief survey of results on the local-global principle (with respect to discrete valuations) for isotropy of quadratic forms over function fields of curves, I will show how to produce counterexamples over function fields of surfaces. The construction involves a special class of quadratic forms of geometric origin whose degeneration locus is a smooth divisor on a smooth model of the function field. This is joint work with R. Parimala and V. Suresh.

A cohomological Hasse principle for fraction fields of two-dimensional local rings (Yong Hu)

Let K be a finite extension of a two-variable Laurent series field $k((x,y))$ and let n be an integer invertible in K. I will explain a proof of the local-global principle for the Galois cohomoloy groups $H^r(K, \mu_n^{\otimes(r-1)})$, $r > 1$. The proof uses only étale cohomology machinery and does not rely on patching techniques. Some applications to quadratic forms will be discussed if time permits.

Arithmetic duality theorems for function fields of curves over local fields and arithmetical applications (Diego Izquierdo)

Let k be a field such as $\mathbb{Q}_p$, $\mathbb{Q}_p((t))$, $\mathbb{C}((t))$ or $\mathbb{C}((t))((u))$. Let X be a smooth projective geometrically integral curve over k, and K the function field of X. I will talk about arithmetic duality theorems for tori over K and then I will focus on some examples and applications of this theorem (weak approximation for tori, local-global principle for central simple algebras or for torsors under linear connected algebraic groups).