ERRATUM TO "THE LOCAL LIFTING PROBLEM FOR ACTIONS OF FINITE GROUPS ON CURVES."

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Abstract

We correct Theorem 1.5 in our paper "The local lifting problem for actions of finite groups on curves," Ann. Sci. Éc. Norm. Supér. (4) 44 (2011), no. 4, 537 - 605.

Résumé

Nous corrigeons le théorème 1.5 de notre article "The local lifting problem for actions of finite groups on curves," Ann. Sci. Éc. Norm. Supér. (4) 44 (2011), no. 4, 537 - 605.

With the notation of [1], the goal of this note is to explain how examples found by B. Weaver in [2] lead to the following corrected form Theorem 1.5 of [1].

Theorem 1.5. *For k algebraically closed of characteristic p, a group G is an almost KGB group for k if and only if it is an almost Bertin group k. Such G are exactly those isomorphic to a group of one of the following kinds:*

i. *A cyclic group.*
ii. *A dihedral group D_{2p^n} of order $2p^n$ for some $n \geq 1$.*
iii. *The alternating group A_4 when $p = 2$.*
iv. *A generalized quaternion group Q_{2^m} of order 2^m for some $m \geq 4$ when $p = 2$.*

The original statement of Theorem 1.5 of [1] had also included the groups $\mathrm{SL}_2(\mathbb{Z}/3)$ and Q_8 when $p = 2$. We had shown that every almost KGB group is on the resulting larger list, and we claimed that the converse holds as well. But examples constructed by B. Weaver in chapter 7 of [2] showed that $\mathrm{SL}_2(\mathbb{Z}/3)$ and Q_8 are not in fact almost KGB groups when $p = 2$, and we thank him for pointing this out.

Theorem 1.5 above and Theorem 1.2 of [1] yield the following (see [2, Prop. 7.2.11]).

Corollary. *For $k = \overline{k}$ of characteristic p, the notions of being a KGB group, a Bertin group, an almost KGB group and an almost Bertin group coincide.*

We now list the results stated in [1] that are not correct and indicate how to correct them.

1. Lemma 15.15 and Corollary 15.16 are not correct. In §15, the field k is assumed to be a quasi-finite field of characteristic p and is not assumed to be algebraically closed (see

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Hypothesis 15.1). The error in Lemma 15.15 arises from the statement just after equation (15.33) that if $\lambda \in k$ then there is a $\zeta \in K$ so $\zeta^2 - \zeta^4 = \lambda$. This is not true, for example, if k has order 2. As a result, Corollary 15.16 does not hold. The error in Lemma 15.15 concerns only the claim that $i_0 = 1$ under the hypotheses of Proposition 15.6(i). The existence of examples in which i_0 is arbitrarily large accounts for why Q_8 and $\mathrm{SL}_2(3)$ are not almost Bertin groups.

2. The proof of Proposition 15.6(i) relied on Corollary 15.16 and is not correct. Proposition 15.6(i) should be changed to the following statement. The integer i_{n-1} is even unless G is the quaternion group of order 8. If i_{n-1} is odd, then the Bertin obstruction of $\phi : G \to \mathrm{Aut}_k(k[[t]])$ does not vanish, and the lower ramification filtration of G is has the form

$$G_0 = \cdots = G_{i_0} \neq G_{i_0+1} = \cdots = G_{3i_0} \neq G_{3i_0+1} = \{e\}$$

where G_{i_0+1} is the center of G. The proof of this result given on page 589 is correct except for the fact that one cannot say that $i_0 = 1$ due to the error in Lemma 15.15.

3. The statement of part (ii) of Proposition 15.6 is correct, but the proof needs to be corrected because it also relied on Corollary 15.16. To fix it, we suppose as in part (ii) of Proposition 15.6 that i_{n-1} is even. One needs to show that not all of hypotheses (i), (ii) and (iii) of Lemma 15.13 can hold. If all these hypotheses hold, then $n = 2$ and $c(1) = 2i_0$, but we cannot say $i_0 = 1$ due to the error in Corollary 15.16. However, Corollary 15.5 shows that i_{n-1} is even if and only $c(n-1)$ is odd. Since $n = 2$ and $c(n-1) = c(1) = 2i_0$ is even, this is impossible since i_{n-1} even. The rest of the proof of part (ii) of Proposition 15.6 now proceeds as in [1].

4. Corollary 15.7 is not correct. It does not follow from work of Serre and Fontaine, and it is not implied by the corrected version of Proposition 15.6. We never use Corollary 15.7 elsewhere in [1].

5. The assertion of Proposition 17.2(i) is correct but the proof should be adjusted in the following way. Instead of referring to Proposition 15.6(iii), one should use the corrected form of Proposition 15.6(i,ii) indicated in items #2 and #3 above. Proposition 17.2(ii) and Corollary 17.3 are not correct because of the examples found by B. Weaver [2] showing that Q_8 and $\mathrm{SL}_2(3)$ are not almost KGB groups for k when $p = 2$. The error in the arguments of [1] concerning Proposition 17.2(ii) and Corollary 17.3 is that Proposition 15.6(i) is not correct in its original form, for the reasons described above. The incorrect proof of Proposition 17.2(ii) relied on the incorrect Proposition 15.6(i) (rather than on Proposition 15.6(iii)), and the proof of Corollary 17.3 is incorrect because if relied on Proposition 17.2(ii). Proposition 17.2(iii) and its proof are correct. Proposition 16.1 holds since the proofs carry through with the correction in item 2 above to part (i) of Proposition 15.6. Note that Lemma 17.5 relies only on part (ii) of Proposition 15.6 rather than on part (i) of Proposition 15.6, and part (ii) of Proposition 15.6 is correct.

6. Theorem 1.5 should exclude $\mathrm{SL}_2(3)$ and Q_8 from the list of almost Bertin groups when $p = 2$.

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