1. Let $A \subseteq B$ be a finite extension of Dedekind domains with fraction field extension $K \subseteq L$, which we assume to be G-Galois. Let P be a non-zero prime ideal of B, and let $\mathfrak{p} = P \cap A$. Assume that B/P is separable over $A/\mathfrak{p}$. Let $\hat{A}, \hat{B}$ be the completions of A, B at $\mathfrak{p}, P$.

a) Show that the ramification groups G_i of $A \subseteq B$ at P are the same as those of $\hat{A} \subseteq \hat{B}$, for $i \geq -1$.

b) Let $\hat{A}'$ be the maximal unramified extension of $\hat{A}$, and let $\hat{B}'$ be the compositum of $\hat{B}$ with $\hat{A}'$. Show that the ramification groups G_i of $\hat{A} \subseteq \hat{B}$, for $i \geq 0$, agree with those of $\hat{A}' \subseteq \hat{B}'$. What about G_{-1}? What can you say about the residue fields of $\hat{A}'$ and $\hat{B}'$?

2. a) Show that the p-adic absolute value on $\mathbb{Q}_p$ extends uniquely to an absolute value on $\bar{\mathbb{Q}}_p$, the algebraic closure of $\mathbb{Q}_p$.

b) Show that $\bar{\mathbb{Q}}_p$ is *not* complete with respect to this absolute value.

c) Prove that the completion of $\bar{\mathbb{Q}}_p$ with respect to this absolute value is an algebraically closed field.

3. Show that Herbrand's theorem implies that $(G/H)_i = G_i/H$ if $H = G_j$ with $j \geq i$.

4. What is the largest n such that there is a degree 2 Galois extension of $\mathbb{Q}$ with $G_n \neq 1$ at some prime? What if you replace $\mathbb{Q}$ by $F(x)$, where F is a finite field?

5. a) Let k be a field of characteristic 2, let $A = k[x]$, and let $A \subset B$ be a finite extension of Dedekind domains whose corresponding fraction field extension $K \subset L$ is Galois of degree 2. Show that there is a complete discrete valuation ring R of characteristic zero whose residue field $R/\mathfrak{m}$ is k, together with a finite extension of integrally closed domains $R[x] \subset \mathcal{S}$, such that the corresponding extension of fraction fields is Galois, and such that $\mathcal{S}/\mathfrak{m}\mathcal{S}$ is isomorphic to B, compatibly with the isomorphism of $R[x]/\mathfrak{m}R[x] = k[x]$ with A.

b) What becomes harder if each 2 is replaced by 3 or some larger integer?

6. Let K be a field and let E be a K-algebra of dimension n. Assume that a group G of order n acts faithfully on E as a K-algebra, such that the set of fixed elements in E is exactly K. We then call E a *G-Galois* K-algebra.

a) Show that to give a field extension L of K which is a G-Galois K-algebra is equivalent to giving a Galois field extension of K together with an isomorphism of $\mathrm{Gal}(L/K)$ with G.

b) If H is a subgroup of G and if L is an H-Galois field extension of K, consider the K-algebra $\mathrm{Ind}_H^G L := L^{(G:H)}$, where the factors are indexed by the right cosets of H in G, and where addition and multiplication are component-wise. Show that there is an action of G on $\mathrm{Ind}_H^G L$ that makes it a G-Galois K-algebra. Describe this algebra in the special cases in which $H = G$ and in which H is trivial. (Doing those cases first might help with the previous part.)

c) Show that if A is a finite abelian group, then the isomorphism classes of A-Galois field extensions of K are in bijection with the surjective homomorphisms $\mathrm{Gal}(K) \to A$; and that the isomorphism classes of A-Galois K-algebras are in bijection with $\mathrm{Hom}(\mathrm{Gal}(K), A)$.

d) Show how the assertion in (c) must be modified if non-abelian groups are considered.