

For each of the following, either give an example, or else prove that none exists.

1. A  $\mathbb{Z}$ -module that is not flat.
2. A non-zero finitely generated module  $M$  over the local ring  $\mathbb{Z}_{(2)}$  such that  $2M = M$ .
3. A commutative ring  $R$ , an  $R$ -module  $M$ , and a surjective homomorphism  $N' \rightarrow N$  of  $R$ -modules such that  $\text{Hom}_R(M, N') \rightarrow \text{Hom}_R(M, N)$  is not surjective.
4. A finitely generated torsion-free  $\mathbb{Z}$ -module that is not projective.
5. A flat  $\mathbb{C}[x]$ -module that is not faithfully flat.
6. A Noetherian ring  $R$  such that  $R[x, y]$  is not Noetherian.
7. A prime ideal in  $\mathbb{C}[x, y, z]/(z^2 - xy)$  of height 3.
8. A maximal ideal in  $\mathbb{Q}[x, y]$  that is not of the form  $(x - a, y - b)$  with  $a, b \in \mathbb{Q}$ .
9. A Dedekind domain  $R$  such that  $R[x]$  is a Dedekind domain.
10. A non-trivial integral ring extension of  $\mathbb{Z}$  that is contained in  $\mathbb{Q}$ .
11. Two unequal field extensions  $E, F$  of  $\mathbb{Q}$ , each of degree 3 and contained in  $\bar{\mathbb{Q}}$ , such that the compositum  $EF$  does not have degree 9 over  $\mathbb{Q}$ .
12. A Galois field extension  $E$  of  $\mathbb{F}_4$  such that  $\text{Gal}(E/\mathbb{F}_4)$  is isomorphic to  $\mathbb{Z}/2 \times \mathbb{Z}/2$ .
13. A Galois field extension of  $\mathbb{C}((x))$  having degree equal to 8.
14. A Galois field extension  $L/K$  of degree 5 such that no non-zero element of  $L$  has trace equal to 0.
15. An irreducible polynomial  $f(x) \in \mathbb{Q}[x]$  whose splitting field has degree 24 over  $\mathbb{Q}$ , such that  $f$  is not solvable by radicals.