Hints for problem 2 on Math 603, spring 2016, problem set 5

For (i) $\Rightarrow$ (ii) and for (iv) $\Rightarrow$ (i), use condition (iii) of problem 1. Also in (i) $\Rightarrow$ (ii), what is the extension to A of a maximal ideal of R?

To deduce the contrapositive of (iv) from (ii), reduce to the case that N is generated by just one element, so that N is a quotient of R by an ideal I that is contained in a maximal ideal $\mathfrak{m}$ of R; then use (ii) to get $\mathfrak{m}A \neq A$ and obtain the conclusion.

So (i) $\Leftrightarrow$ (ii) $\Leftrightarrow$ (iv).

The implication (iii) $\Rightarrow$ (ii) is easy. For the converse, use (ii) $\Rightarrow$ (i) to show that $A_{\mathfrak{p}} := A \otimes_R R_{\mathfrak{p}}$ is faithfully flat over $R_{\mathfrak{p}}$ for any prime ideal $\mathfrak{p} \subset R$, and then apply the implication (i) $\Rightarrow$ (ii) to $A_{\mathfrak{p}}$ and $R_{\mathfrak{p}}$.