For each of the following, either give an example, or else explain why none exists.

1. A finite group that is solvable but not nilpotent.
2. A group of order 40 with no subgroup of order 4.
3. A non-abelian group of order 55.
4. A non-split short exact sequence of finite groups.
5. A finite abelian group whose automorphism group is non-abelian.
6. A simple group of order 250.
7. A non-principal ideal in an integral domain.
8. A non-trivial proper left ideal in $M_3(\mathbb{F}_5)$ (i.e. 3×3 matrices over $\mathbb{F}_5 = \mathbb{Z}/5\mathbb{Z}$).
9. A unique factorization domain of Krull dimension 4.
10. A maximal ideal in $\mathbb{Q}[x, y]/(x^3 + y^3 - 1)$ whose residue field is not isomorphic to $\mathbb{Q}$.
11. A commutative ring that is not an integral domain, and whose nilradical is trivial.
12. A set of eight linearly independent vectors in $\mathbb{R}^3 \otimes \mathbb{R}^4$.
13. A natural isomorphism $V^* \otimes_F W \to \mathrm{Hom}(V, W)$, where V, W are finite dimensional vector spaces over a field F.
14. A non-zero homomorphism $V \to W$ of finite dimensional vector spaces over a field F together with a non-zero finite dimensional F-vector space Z such that the induced map $\mathrm{Hom}(Z, V) \to \mathrm{Hom}(Z, W)$ is the zero map.
15. A commutative ring R and an R-module M that does not have a basis over R.