1. a) How many ways can a regular tetrahedron be inscribed in a cube? (Here, inscribing is required to be "tight". That is, the vertices of the tetrahedron are required to be placed at vertices of the cube, and the edges of the tetrahedron are required to be placed along diagonals of faces of the cube.)

b) What homomorphisms between the symmetry groups of a solid regular tetrahedron and of a solid cube, if any, are induced by the inscribing of a tetrahedron in a cube? Describe the kernel and image of any such homomorphism.

c) Repeat parts (a) and (b) with a cube and a regular dodecahedron, rather than with a regular tetrahedron and a cube.

2. a) Show that the symmetries of a solid regular tetrahedron form the alternating group A_4. [Hint: What happens to the vertices of the tetrahedron, under a symmetry?]

b) Show that the symmetries of a solid cube form the symmetric group S_4. [Hint: What happens to the long internal diagonals of the cube, under a symmetry?]

c) Show that the symmetries of a solid regular dodecahedron form the alternating group A_5. [Hint: What happens to the various cubes that can be inscribed in the dodecahedron, under a symmetry?]

d) Are the orders of these groups consistent with your answers to Problem 1? [Hint: Be careful here.]

3. a) Which of the following groups can be written as a direct product of cyclic p-groups (possibly for various p's)? $\mathbb{Z}/10 \times \mathbb{Z}/15$, D_5, A_4, Q. For those which can be, do so.

b) Find all the Sylow subgroups of the above groups, and verify that the Sylow theorems hold there.

4. Let G' be the commutator subgroup of a finite group G, and let N be a subgroup of G. Show that the following are equivalent:
(i) N is normal in G, and G' is contained in N.
(ii) N is the kernel of a surjective homomorphism from G to an abelian group.

5. Let G be a p-group, let G' be its commutator subgroup, and let Φ be its Frattini subgroup.

a) Show that if N is a maximal subgroup of G, then N contains G'. [Hint: Problem 4.]

b) Deduce that G' is a subgroup of Φ.

c) Conclude that G/Φ is abelian.