Math 602 | Sample Exam | Fall 2015

For each of the following, either give an example, or else prove that none exists.

1. A non-abelian group of order 55.
2. A group of order 54 with no subgroup of order 9.
3. A finite group that is solvable but not nilpotent.
4. A non-split short exact sequence of finite groups.
5. A finite abelian group whose automorphism group is non-abelian.
6. A non-trivial proper left ideal in $M_3(\mathbb{F}_5)$ (i.e. 3×3 matrices over $\mathbb{F}_5 = \mathbb{Z}/5\mathbb{Z}$).
7. A maximal ideal in $\mathbb{Q}[x, y]/(x^3 + y^3 - 1)$ whose residue field is not isomorphic to $\mathbb{Q}$.
8. A unique factorization domain of Krull dimension 4.
9. A commutative ring whose nilradical is unequal to its Jacobson radical.
10. An ideal I in an integral domain R such that I is radical but not prime.
11. A set of eight linearly independent vectors in $\mathbb{R}^3 \otimes \mathbb{R}^4$.
12. A natural isomorphism $V^* \otimes_F W \to \mathrm{Hom}(V, W)$, where V, W are finite dimensional vector spaces over a field F.
13. An injective homomorphism $\bigwedge^2 \mathbb{R}^4 \to \mathbb{R}^5$.
14. A non-zero homomorphism $V \to W$ of finite dimensional vector spaces over a field F together with a non-zero finite dimensional F-vector space Z such that the induced map $\mathrm{Hom}(Z, V) \to \mathrm{Hom}(Z, W)$ is the zero map.
15. A quadratic form q over a field K of characteristic zero such that q is both isotropic and regular.