Math 503 Problem Set #14 Due the week of April 23, 2018, in class.

Reminder: There will be an exam in class on Wednesday, April 25. This problem set is a sample exam. Those who complete this and submit it in class on Monday, April 23, will be given extra credit. As on the actual exam, you should do all the problems, showing your work and explaining your assertions.

Read Artin, Chapter 16, sections 8-12. Review Chapter 12, sections 3 and 4, and Chapters 13-16.

Part I:

1. Find all positive integers n such that all abelian groups of order n are isomorphic.

2. Find the degree of the field $\mathbb{Q}[\sqrt{7+\sqrt{7}}]$ over $\mathbb{Q}$. Explain.

3. Find a finite extension of $\mathbb{Q}$ that is not a splitting field of a polynomial over $\mathbb{Q}$.

4. Find all subfields of a field of 64 elements.

Part II:

5. Determine which of the following are free modules over $\mathbb{Z}$: $\mathbb{Z}[\sqrt{3}]$, $\mathbb{Z}/3\mathbb{Z}$, $\mathbb{Z}[1/3]$, $3\mathbb{Z}$. For those that are free, determine the rank.

6. Determine whether the rings $\mathbb{Q}[\sqrt{3}]$ and $\mathbb{Q}[\sqrt[3]{3}]$ are isomorphic. Also determine whether they are fields.

7. Let I be an ideal of $\mathbb{Z}[\sqrt{23}]$. Show that I is finitely generated.

8. Let $K = \mathbb{Q}[\sqrt{3}, \sqrt{7}]$.
a) Find all automorphisms of K, and determine what group they form.
b) Find all fields F such that $\mathbb{Q} \subseteq F \subseteq K$, and determine which are Galois extensions of $\mathbb{Q}$.