Math 503 Problem Set #13 Due the week of April 16, 2018, in lab.

In Artin, Chapter 16, read sections 1-7.

1. From Artin, Chapter 15, do problem 8.2 (pages 472-476). From Artin, Chapter 16, do these problems (pages 505-512): 3.1, 3.2, 4.1(b), 5.1(b,c), 6.2, 7.3.

2. Let p be a prime number and let $E = \mathbb{Q}[\zeta_p]$, where ζ_p is a primitive pth root of unity. Let $F = \mathbb{Q}$.

  a) Find the degree and the Galois group G of the field extension $F \subset E$.
  b) Determine if the extension is separable and normal.
  c) Find the fixed field of G in E.
  d) Is the extension Galois?

3. Let $L = \mathbb{Q}[\sqrt{2+\sqrt{2}}]$.

  a) Show that L is Galois over $\mathbb{Q}$, and that its Galois group is cyclic of order 4. [Hint: PS 11, #4.]
  b) Show that L is not the splitting field over $\mathbb{Q}$ of a polynomial $x^4 - a$ for some $a \in \mathbb{Q}$.
  c) What happens to part (a) if we instead consider the field $\mathbb{Q}[\sqrt{3+\sqrt{3}}]$?

4. Let $K = \mathbb{C}(x)$ and $L = K[\sqrt[6]{x}]$. Show that L is Galois over K, find the Galois group, find all intermediate fields M (i.e. fields with $K \subset M \subset L$), and find all the Galois groups $\mathrm{Gal}(L/M)$ and $\mathrm{Gal}(M/K)$ for these fields M. Verify in this example that M is Galois over K if and only if $\mathrm{Gal}(L/M)$ is a normal subgroup of $\mathrm{Gal}(L/K)$, and that $\mathrm{Gal}(M/K)$ is then the quotient of these two groups.

5. Redo problem 4 above for the fields $K = \mathbb{Q}$ and $L = K[\zeta_3, \sqrt[3]{2}]$, where ζ_3 is a primitive cube root of unity.