Math 502 Problem Set #4 Due week of Sept. 25, 2017, in lab.

Read Artin, Chapter 2, sections 7-8.

1. From Artin, Chapter 2, do these problems (pages 69-77):
5.1, 5.2, 6.4, 7.1.

2. Which of the following relations are reflexive? symmetric? transitive? equivalence relations?

i) On $\mathbb{Z}$, $a \sim b$ if $a+b$ is even.
ii) On $\mathbb{Z}$, $a \sim b$ if ab is even.
iii) On $\mathbb{R}$, $a \sim b$ if $ab > 0$.
iv) On $\mathbb{R}^\times$, $a \sim b$ if $ab > 0$.

3. Find the logical flaw in the following argument, which purports to prove the (false) assertion that if a relation is symmetric and transitive then it must be reflexive and hence be an equivalence relation: "If $a \sim b$ then $b \sim a$ by symmetry; and so $a \sim a$ by transitivity. So the relation is reflexive." Also, give an example of a relation that shows that the conclusion of the argument need not be true. [You may use an example from problem 2.]

4. Which of the following are subgroups H of the given group G? Which are normal subgroups?

a) $H = \{\text{permutations that take 1 to 1}\}$, $G = S_4$.
b) $H = \{\text{transpositions in } S_5\} \cup \{\text{id}\}$, $G = S_5$.
c) $H = \mathrm{SL}_2(\mathbb{R})$, $G = \mathrm{GL}_2(\mathbb{R})$.

5. Consider the map $\phi : D_4 \to \mathbb{Z}/2\mathbb{Z}$ that takes each rotation to 0 and each flip to 1.

a) Show that ϕ is a surjective homomorphism, and find its kernel.
b) Describe the fibers of ϕ, and show that these fibers are the same as the cosets of the kernel of ϕ.