Math 502 Problem Set #1 Due week of Sept. 4, 2017, in lab.

Review Artin, Chapter 1, especially sections 4-6.

For each of the following sets, determine whether it is a group, a ring, a field, or a vector space. (Some may be more than one.) Say under what operation(s) it forms such a structure (choose from among addition, multiplication, composition). Also give the identity element(s), and describe the inverses if they exist. In the case of groups and rings say whether they are commutative (abelian, for groups).

Note: Some of these problems are tricky.

1. The set of positive real numbers.

2. The set of symmetries of the unit circle.

3. The set of series $\sum_{i=0}^{\infty} a_i x^i$, with a_i real.

4. The set of series $\sum_{i=-\infty}^{\infty} a_i x^i$, with a_i real.

5. The set of 2×2 matrices whose entries are integers.

6. The set of 2×2 matrices whose entries are positive integers.

7. The set of real numbers of the form $a + b\sqrt{2}$, where a, b are rational numbers.

8. The set of real numbers of the form $a + b\sqrt[3]{2}$, where a, b are rational numbers.

9. The set of real polynomials $f(x)$ such that $f^{(n)}(0) = 0$. (This question depends on the value of n, which is a fixed non-negative integer. Here $f^{(n)}$ denotes the n-th derivative.)

10. The set of strictly increasing continuous real-valued functions f on the closed interval $[0, 1]$ such that $f(0) = 0$ and $f(1) = 1$. (A function f is *strictly increasing* if $a < b$ implies $f(a) < f(b)$.)