Math 502, fall 2017 Warm-up Problem Set Discussed in lab, week of Aug. 28, 2017

1. Determine which of the following is a real vector space. For each one that is a real vector space, determine if it is finite dimensional; and if so, find a basis. For each one that is not a real vector space, find which part of the definition is not satisfied. Among those listed below that are finite dimensional vector spaces, determine if any two are isomorphic.

a) $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = z^2\}$, under the usual addition and scalar multiplication in $\mathbb{R}^3$.

b) $\{(x, y) \in \mathbb{R}^2 \mid y \in \mathbb{Q}\}$, under the usual addition and scalar multiplication in $\mathbb{R}^2$. (Here $\mathbb{Q}$ is the set of rational numbers.)

c) The set of 2×2 real symmetric matrices, under addition and scalar multiplication of matrices.

d) The set of 2×2 real orthogonal matrices, under addition and scalar multiplication of matrices.

e) Fix $c \in \mathbb{R}$. Take the set V_c of C^∞-functions $f : \mathbb{R} \to \mathbb{R}$ such that $f''(x) + f(x) = c$, under addition of functions and multiplication by scalars. (This is a different problem for each value of c.)

f) $V = \{(x, y) \in \mathbb{R}^2 \mid x, y > 0\}$, under the vector addition law given by $(x_1, y_1) + (x_2, y_2) := (x_1x_2, y_1y_2)$ and the scalar multiplication law given by $c \cdot (x, y) := (x^c, y^c)$ for $c \in \mathbb{R}$ and $(x, y) \in V$.

2. Determine which of the following is a (well-defined) linear transformation of real vector spaces. Among those that are, determine which are isomorphisms.

a) The map from the set of 2×3 real matrices to the set of 3×2 real matrices, taking each matrix to its transpose.

b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (x^2, y^2)$.

c) $V = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is } C^\infty\}$, $T : V \to V$ is given by $T(f) = g$ where $g(x) = f(x) \sin(x)$.

d) $T : \mathbb{C}^2 \to \mathbb{C}^2$ given by $T(z_1, z_2) = (\bar{z}_1, \bar{z}_2)$ (where bar denotes complex conjugation). Also determine if this is a linear transformation of complex vector spaces.

e) Let V be the set of C^∞ functions $f : \mathbb{R} \to \mathbb{R}$, and let W be the set of equivalence classes of C^∞ functions $f : \mathbb{R} \to \mathbb{R}$, where we declare f, g to be equivalent if $f - g$ is a constant function. Define $T : V \to W$ by $T(f) =$ the equivalence class of some antiderivative of f.