Math 371 Problem Set #14 Due week of April 28, 2008, in lab.

Note: There is no lab on Thursday, May 1; the last lab is on Tuesday, April 29, and all are welcome to attend. This assignment may be handed in either at the last lab or at the last class, on Monday, April 28.

Reminder: The final exam will take place on Tuesday, May 13, from noon to 2pm, in DRL A4.

Read Herstein, Chapter 5, sections 6-8.

1. From Herstein, do these problems:

a) Section 2.12, page 103: #20, in the case $n = 2$.
b) Section 2.13, page 109: #13. [Hint: try a group G of order 8.]
c) Section 2.14, page 115: #4(d). Also determine how many such groups there are (up to isomorphism).
d) Chapter 2 supplementary problems, page 116: #10.
e) Section 5.6, pages 249-250: #6, 18. [Note: In this section of Herstein, fields are assumed to have characteristic 0.]

2. Which of the following groups are isomorphic? $\mathbb{Z}/6$, $6\mathbb{Z}$, S_6, S_3, D_3, $\mathbb{Z}/2 \times \mathbb{Z}/3$, $\mathbb{Z}/2 \times \mathbb{Z}/2$, $\mathbb{Z}/4$, $\mathbb{Z}/7$, $(\mathbb{Z}/7)^\times$. Explain.

3. a) Show that every group of order $259 = 7 \cdot 37$ is cyclic.
b) Do the same for groups of order 1295.

4. Find all groups of order 34. [Hint: Are any Sylow subgroups normal? If x has order 17 and y has order 2, what is yxy^{-1}?]

5. Find the Galois groups $\mathrm{Gal}(\mathbb{Q}(\sqrt{7})/\mathbb{Q})$ and $\mathrm{Gal}(\mathbb{Q}(\zeta_7)/\mathbb{Q})$, where $\zeta_7 = e^{2\pi i/7}$. In particular, determine the orders of these groups and whether they are abelian and whether they are cyclic.