Math 371                    Problem Set #13              Due week of April 21, 2008, in lab.

Read Herstein, Chapter 2, sections 12-14.

1. From Herstein, Chapter 2, do these problems:

   a) Section 2.10, page 81: #11. [Hint: Take products and conjugates, and use that every permutation is a product of transpositions.]
   b) Section 2.11, pages 90-91: #8, 18.
   c) Section 2.12, page 102: #6, 11(a).
   d) Section 2.13, page 108: #6, 7.
   e) Section 2.14, page 115: #4(a).

2. Show that the dihedral group $D_4$ is not isomorphic to the quaternion group.

3. Let $G_0 = \mathbb{Z}/12$, let $G_1 = \mathrm{Aut}(G_0)$, and let $G_2 = \mathrm{Aut}(G_1)$. For each $i$ find the order of $G_i$, and determine whether $G_i$ is abelian and whether it is cyclic.

4. Find the smallest integer $n > 100$ such that there are at least two non-isomorphic abelian groups of order $n$. How many (non-isomorphic) abelian groups of this order are there?

5.   a) Show that every group of order 35 is cyclic.
     b) Find all groups of order 175 and show that all such groups are abelian.