Math 371 Problem Set #12 Due week of April 14, 2008, in lab.

Read Herstein, Chapter 2, sections 9-11.

1. From Herstein, Chapter 2, do these problems:

a) Section 2.7, pages 65-66: #15-17. [Hint for #15 and #17: use the first isomorphism theorem.]

b) Section 2.8, page 70: #5.

c) Section 2.9, pages 74-75: #4.

d) Section 2.10, pages 80-81: #1(b), 3(b), 5, 10.

e) Section 2.11, page 90: #7.

2. Let G' be the commutator subgroup of a group G (see problem 4 on PS #11).

a) Show that G/G' is well defined and is abelian.

b) Find G/G' if $G = \mathbb{Z},\ \mathbb{Z}/5,\ S_3,\ D_4$.

3. Show that $SL_2(\mathbb{R})$ is a normal subgroup of $GL_2(\mathbb{R})$. Which familiar group is isomorphic to $GL_2(\mathbb{R})/SL_2(\mathbb{R})$? [Hint: use the first isomorphism theorem.]

4. a) Let $\phi : G \to H$ be a homomorphism of groups, and let N be a normal subgroup of H. Show that $\phi^{-1}(N)$ is a normal subgroup of G.

b) What does part (a) tell us when N is the trivial subgroup of H?

5. Let G be a finite group, let $g \in G$, and let $h = g^r$.

a) Show that if $o(g)$ is relatively prime to r then $o(g) = o(h)$.

b) Show that if $r|o(g)$, then $o(h) = o(g)/r$.

c) Show that in general, $o(h) = o(g)/d$, where d is the greatest common divisor of r and $o(g)$.