Math 371 Problem Set #11 Due week of April 7, 2008, in lab.

Read Herstein, Chapter 2, sections 7-8.

1. From Herstein, Chapter 2, do these problems:

a) Section 2.5, page 47: #13.

b) Section 2.6, pages 53-54: #3, 5, 11, 12, 14. [Hint for #12: Show $nmn^{-1}m^{-1}$ is in both N and M.] [Hint for #14: Pick a group G of order 8.]

c) Section 2.7, pages 64-65: #1, 6. [Hint for #6: Send each element on the right hand side to the corresponding coset on the left hand side.]

d) Section 2.8, page 70: #1, 4.

2. The *quaternion group* $H = \{\pm 1, \pm i, \pm j \pm k\}$ has an operation denoted by multiplication and has identity element 1. The element -1 is in the center (i.e. commutes with everything), and has order 2. Also $i^2 = j^2 = k^2 = -1$ and $(-1)i = -i$, $(-1)j = -j$, $(-1)k = -k$, $ij = k$, $jk = i$, $ki = j$, $ji = -k$, $kj = -i$, $ik = -j$.

a) Explicitly write out the full multiplication table for H (an 8×8 matrix).

b) Find all the subgroups of H. Which are abelian? Which are cyclic?

3. Suppose $\phi : G \to H$ is a surjective homomorphism with kernel N. For any subgroup $K \subset H$, define $f(K) = \phi^{-1}(K) = \{g \in G \,|\, \phi(g) \in K\}$. Show that f defines a bijection from the set of subgroups of H to the set of subgroups of G that contain N. Also show that $f(J) \subset f(K)$ if and only if $J \subset K$.

4. Define the *commutator subgroup* G' of a group G to be the subgroup of G generated by the elements of the form $aba^{-1}b^{-1}$ for all $a, b \in G$.

a) Show that G' is a normal subgroup of G.

b) Find G' if $G = \mathbb{Z},\ \mathbb{Z}/5,\ S_3,\ D_4$.