Math 371 Problem Set #10 Due week of March 31, 2008, in lab.

Reminder: Exam #2 will take place in class on Wednesday, April 2, on the material covered in class up through Friday, March 28, and the related material on homeworks.

Read Herstein, Chapter 2, sections 4-6.

1. From Herstein, Chapter 2, do these problems:

a) Section 2.5, pages 46-48: #1, 4, 14, 16, 27.

b) Section 2.6, page 53: #2, 4. [Hint for #2: what are the cosets of $H$?]

2. If $G$ is a group, define the *center* of $G$ to be the subset

$$Z(G) = \{g \in G \,|\, \forall h \in G,\ gh = hg\} \subset G.$$

a) Show that $Z(G)$ is a subgroup of $G$.
b) Describe $Z(G)$ for each of the following groups: $\mathbb{R}, S_3, D_4, Z(D_4), GL_2(\mathbb{R})$.
c) For each of the groups in (b), find $|G|$ and $|Z(G)|$.

3. a) Let $G = GL_n(\mathbb{R})$ under matrix multiplication. Find a subgroup $H \subset G$ such that

$$A \equiv B \,(\mathrm{mod}\, H) \Leftrightarrow \det(A) = \det(B)$$

for $A, B \in G$.

b) Let $G = M_n(\mathbb{R})$ under matrix addition. Find a subgroup $H \subset G$ such that

$$A \equiv B \,(\mathrm{mod}\, H) \Leftrightarrow \mathrm{trace}(A) = \mathrm{trace}(B)$$

for $A, B \in G$.

4. Find all the finite subgroups of the multiplicative group $\mathbb{C}^\times = \mathbb{C} - \{0\}$. Justify your assertion.