Math 371   Problem Set #9   Due week of March 24, 2008, in lab.

Read Herstein, Chapter 5, sections 4-5; Chapter 2, sections 1-3.

1. From Herstein, do these problems:
    a) Chapter 5, section 4, page 232: #9.
    b) Chapter 2, section 3, pages 35-37: #2, 8, 9, 11, 21, 22, 24, 25.

2. Which of the following are groups? For those that are: are they abelian? how many elements do they have?
    a) The set of symmetries of an isosceles right triangle, under composition.
    b) The set of symmetries of a square, under composition.
    c) The set of invertible 7×7 real matrices with positive determinant, under matrix multiplication.
    d) $GL_3(\mathbb{R})$, under matrix addition.
    e) The power set of $\mathbb{Z}$, under union.

3. Find all the subgroups of $\mathbb{Z}$. Do the same for $\mathbb{Z}/n$.

4.   a) For which integers n does the dihedral group D_5, of symmetries of the regular pentagon, contain a subgroup having exactly n elements?
    b) Redo part (a), with D_5 replaced by $\mathbb{Z}/18$.