Math 371 Problem Set #2 Due week of Jan. 28, 2008, in lab.

Review Herstein, Chapter 1, section 3.

1. From Herstein, Chapter 1, do these problems:

a) Section 1.2, page 17: #13.

b) Section 1.3, page 24: #6 [Hint: Show that r_n divides both a and b, and also that any integer that divides both a and b must divide r_n], #7, #8 [For #8, also use this to find all prime numbers between 150 and 160].

2. Which of the following are (partial) order relations? Which are total order relations? For which of the total order relations is the set well ordered?

i) On the set of subspaces of a certain vector space V, the relation $W_1 \subset W_2$.
ii) On the set of lines in $\mathbb{R}^3$, the relation of the lines not intersecting.
iii) On the set $\mathbb{C}$, the relation that $|a| \leq |b|$.
iv) On the set of positive real numbers, the relation $a \leq b$.
v) On the set of integers ≥ -6, the relation $a \leq b$.
vi) On the set $\{0, 1, 1/2, 1/3, 1/4, \ldots\}$, the relation $a \leq b$.
vii) On the set of non-negative integers, the relation that *either* $0 < a \leq b$ *or* $b = 0$ (and a is arbitrary).

3. Show that there is no rational number whose square is 2; i.e. that there is *no* non-zero solution in $\mathbb{Z}$ to the equation $x^2 = 2y^2$. [Hint: If $\sqrt{2} = x/y$, we may choose x, y relatively prime. Are they even or odd?]

4. a) Show that there are infinitely many prime numbers. [Hint: Consider which primes divide $n! + 1$.]

b) Show that there are arbitrarily large gaps between consecutive prime numbers. [Hint: Consider which primes divide the numbers $n! + 2, n! + 3, n! + 4, \ldots$.]