Math 370 Sample Exam #2 March, 2014

Instructions: This exam consists of five problems. Do all five, showing your work and explaining your assertions. Allow yourself 50 minutes. Each problem is worth 10 points, for a total of 50 points.

1. Let A, B, C be $n \times n$ matrices over a field F, with C invertible and $B = C^{-1}AC$. Show that A and B have the same rank and have the same nullity.

2. Let $V = \mathbb{R}^3$, with basis i, j, k, and let x, y, z be the dual basis of V^*. Find a basis for the annihilator of the subspace of V spanned by $3i - j$.

3. Define $\phi : \mathbb{R}[x] \to \mathbb{R}[x]$ by $f(x) \mapsto (x^2 + 1)f(x)$. Also, define $\psi : \mathbb{R}[x] \to \mathbb{R}[x]$ by $f(x) \mapsto f(x)^2$. Determine whether ϕ and ψ are linear transformations, and whether they are algebra homomorphisms.

4. Let V be the real vector space of polynomials $f(x) \in \mathbb{R}[x]$ of degree at most 4. Define $T : V \to \mathbb{R}^4$ by $T(f) = (f(1), f(2), f(3), f(4))$. Find the kernel and image of T.

5. a) Is there a linear transformation of $\mathbb{R}$-vector spaces $T : \mathbb{R}^2 \to \mathbb{R}^2$ whose kernel is

$$\{(x, y) \in \mathbb{R}^2 \mid x = 2y\}?$$

If so, find one. If there isn't one, explain why.

b) Is there a homomorphism of $\mathbb{R}$-algebras $T : \mathbb{C} \to \mathbb{C}$ whose kernel is

$$\{x + iy \in \mathbb{C} \mid x, y \in \mathbb{R},\ x = 2y\}?$$

If so, find one. If there isn't one, explain why.