Math 370 | Sample Exam #1 | February, 2014.

Instructions: This exam consists of five problems. Do all five, showing your work and explaining your assertions. Allow yourself 50 minutes. Each problem is worth 10 points, for a total of 50 points.

1. Is $\{(x, y) \in \mathbb{R}^2 \mid xy = 0\}$ a subspace of $\mathbb{R}^2$? Justify your assertion.

2. Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ whose image (range) consists of the scalar multiples of the vector $(1, 3)$. Explain.

3. Let V be the real vector space consisting of all 2×2 real matrices. Let $W \subset V$ consist of the symmetric matrices in V. Determine whether W is a subspace of V. If it is, find its dimension and find a basis of W.

4. Let S, T be finite subsets of $\mathbb{R}^n$, with $S \subset T$. Suppose that S spans $\mathbb{R}^n$ and that T is linearly independent. Show that $S = T$.

5. Consider the vectors $d_1 = (2, 1)$ and $d_2 = (0, 2)$ in $\mathbb{R}^2$. Show that d_1, d_2 form a basis of $\mathbb{R}^2$, and find the coordinates of the vector $(2, 0)$ in this basis.