Math 370 Problem Set #14 Due April 28, 2014, in lecture.

Note: Those who would like an extension on this assignment may hand it in to their TA no later than noon on Friday, May 2.

Reminder: The final exam will be held on Wednesday, May 7, from 9-11 am, in Skirkanich Auditorium.

Read Hoffman and Kunze, Chapter 8, Sections 1-5.

1. From Hoffman and Kunze, Chapter 6, do these problems: Page 198, #2,5,6.

2. From Hoffman and Kunze, Chapter 8, do these problems: Page 276, #9, 12. [Hint for #12: Define $T : V \to \mathbb{R}^n$ by $\alpha \mapsto ((\alpha|\alpha_1), \ldots, (\alpha|\alpha_n))$. Is T an isomorphism?] Pages 288-289, # 1, 3. Page 298, #1. Page 310, #8. Page 317, #9. [Hint for #9: Consider problem 6 of PS 13.]

3. For each of the following matrices, find the characteristic polynomial and the minimal polynomial; determine whether the matrix is similar to a real diagonal matrix and whether it is similar to a real triangular matrix; also whether it is similar to a complex diagonal or triangular matrix. In each case find the diagonal matrix it is similar to, if it exists.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

4. Let V be an inner vector space.

a) Show that if W is a subspace of V, then $W \subset (W^\perp)^\perp$.

b) Show that $W = (W^\perp)^\perp$ if V is finite dimensional. [Hint: If $\dim V = n$ and $\dim W = d$, then what is $\dim W^\perp$? $\dim (W^\perp)^\perp$?]

5. In $\mathbb{R}^4$, let V be the span of $(1,0,1,0)$ and $(1,1,3,1)$.

a) Find an orthonormal basis of V. [Hint: Gram-Schmidt.]

b) Find the point on V closest to $(1,2,3,4)$.

c) Express $(1,2,3,4) = v_1 + v_2$ explicitly, where $v_1 \in V$ and $v_2 \in V^\perp$.

d) Find an orthonormal basis of $V^\perp$.

6. Over $\mathbb{R}$, which of the following matrices have an orthonormal basis of eigenvectors? What about over $\mathbb{C}$? Also, which of these matrices are symmetric? orthogonal? Hermitian? unitary?

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$