Math 370 Problem Set #12 Due week of April 14, 2014, in lab.

Read Hoffman and Kunze, Chapter 5 (sections 5 and 7 are optional).

1. From Hoffman and Kunze, Chapter 5, do these problems:
Page 148-149, #3,5. Page 155, #2, 7. Page 162, #1 (just the second matrix), 2(a), 4.

2. Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & c \end{pmatrix} \in M_n(F)$ where $c \in F$ is a scalar.

a) Find A^{-1}, using row reduction.

b) *Using part* (a), determine for which $c \in F$ there is an inverse for A.

c) Compute the determinant of A.

d) *Using part* (c), determine for which $c \in F$ there is an inverse for A, and compute A^{-1} using the formula for inverses in terms of determinants. Does this agree with your answers to parts (a) and (b)?

3. Let A be a 3×2 matrix and let B be a 2×3 matrix. Find $\det(AB)$. Explain your answer, and explain the connection to problem 3 on Problem Set #6. Can anything be said about $\det(BA)$?

4. Recall that if F if a field, then $\mathrm{GL}_n(F)$ consists of the invertible matrices in $\mathrm{M}_n(F)$. Consider the following subsets of $\mathrm{GL}_n(F)$: $\mathrm{SL}_n(F)$ consists of the matrices with determinant equal to 1. $\mathrm{GL}_n^+(F)$ consists of the matrices with positive determinant. $\mathrm{GL}_n^-(F)$ consists of the matrices with negative determinant. $\mathrm{O}_n(F)$ consists of the matrices A such that $AA^t = I$ (orthogonal matrices). $\mathrm{S}_n(F)$ consists of the matrices $A \in \mathrm{GL}_n(F)$ such that $A = A^t$ (symmetric invertible matrices).

a) Show that $\mathrm{GL}_n(F)$ is a group under matrix multiplication.

b) Determine which of the subsets $\mathrm{SL}_n(F)$, $\mathrm{GL}_n^+(F)$, $\mathrm{GL}_n^-(F)$, $\mathrm{O}_n(F)$, $\mathrm{S}_n(F)$ of $\mathrm{GL}_n(F)$ are groups under matrix multiplication (and hence subgroups of $\mathrm{GL}_n(F)$).

5. Consider the system of linear equations $AX = B$, where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ -3 & 2 & 0 \\ 2 & 0 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}.$$

Solve this system in three different ways:

a) By row reduction on the augmented matrix $(A \,|\, B)$.

b) By finding A^{-1} and writing $X = A^{-1}B$.

c) By Cramer's Rule.