Math 370 Problem Set #9 Due week of March 24, 2014, in lab.

Read Hoffman and Kunze, Chapter 3, Section 7, and Chapter 4, Sections 1-3.

1. From Hoffman and Kunze, Chapter 3, do these problems:
Page 111, #1. Pages 115-116, #1-3, 7.

2. From Hoffman and Kunze, Chapter 4, do these problems:
Pages 122-123, #1(a), 4-6. Pages 126-127, #5, 6. [Hint for #6: what is $L(1)$? $L(x)$?]

3. a) Show that if $T : V \to W$ is a surjective linear transformation of finite dimensional vector spaces with kernel N, then $\dim(V/N) = \dim(V) - \dim(N)$. [Hint: Consider $\dim(W)$.]

b) Illustrate this with the example $V = \mathbb{R}^3$, $W = \mathbb{R}$, $T(x, y, z) = x + y + z$.

4. Let $T : V \to W$ and $S : W \to Z$ be linear transformations of finite dimensional vector spaces.

a) Show that $T^t = 0$ if and only if $T = 0$.

b) Show that $(S \circ T)^t = T^t \circ S^t$, and deduce that if $S \circ T = 0$ then $T^t \circ S^t = 0$. [You can do this either using the linear transformations or the corresponding matrices.]

c) Show that if T is surjective then T^t is injective. [Hint: What is the kernel of T^t?]

d) Show that if T is injective then T^t is surjective. [Hint: Pick a basis B of V, and show that $T(B)$ extends to a basis of W.]

e) Conclude that T is an isomorphism if and only if T^t is an isomorphism.

5. Let F be a field.

a) What does the first isomorphism theorem for groups say about the trace map $\mathrm{M}_2(F) \to F$? (Here $\mathrm{M}_2(F)$ consists of the 2×2 matrices over F, viewed as a group under addition.)

b) What does it say about the determinant map $\mathrm{GL}_2(F) \to F^\times$? (Caution: What are the operations on these groups?)