Math 370 Problem Set #5 Due week of Feb. 17, 2014, in lab.

Reminder: Exam #1 will take place in class on Monday, February 17, on the material covered in class up through Friday, February 14, and on homeworks up through PS 5. Sample Exam 1 is available on the course web page.

Read Hoffman and Kunze, Chapter 3, Section 2.

1. From Hoffman and Kunze, Chapter 3, do these problems:
Pages 73-74, #2,8,12. Page 83, #1,5.

2. Let $S : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation taking $(x, y) \in \mathbb{R}^2$ to $(a, b, c) \in \mathbb{R}^3$ whenever

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}.$$

Find the kernel and image of $S$, describing them both geometrically and in terms of equations.

3. Let $V$ be a real vector space, and suppose that $S : V \to \mathbb{R}$ and $T : V \to \mathbb{R}$ are linear transformations. Define $P : V \to \mathbb{R}^2$ by $P(v) = (S(v), T(v))$.

a) Show that $P$ is a linear transformation.

b) Find the kernel of $P$ in terms of the kernels of $S$ and $T$.

4. For each of the following, either give an example or explain why no such example exists:

a) A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ whose kernel is one-dimensional.

b) A linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ whose kernel is trivial.

c) A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ taking $(1, 2, 3)$ to $(4, 5)$.

5. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. Show that the kernel of $T$ is contained in the kernel of $T^2 = T \circ T$. Can the two kernels ever be equal? Can they ever be unequal? Explain (with examples).