Math 370 Problem Set #4 Due week of Feb. 10, 2014, in lab.

Read Hoffman and Kunze, Chapter 2, Sections 4-6, and Chapter 3, Section 1.

1. From Hoffman and Kunze, Chapter 2, do these problems:
Pages 48-49, #6-7. Page 55, #2. Page 66, #3.

2. From Hoffman and Kunze, Chapter 3, do these problems:
Pages 73-74, #1,4,5,9,10.

3. Which of the following are linear transformations?

a) $T : \mathbb{R}^3 \to \mathbb{R}^4$ sending $(x, y, z) \mapsto (x - y, y - z, z - x, 0)$.

b) $M : \mathbb{R}^2 \to \mathbb{R}$ sending $(x, y) \mapsto xy$. (Here we view $\mathbb{R} = \mathbb{R}^1$.)

c) $R : Z \to Z$ (where Z is the vector space of sequences of real numbers) sending $(a_1, a_2, a_3, \ldots) \mapsto (0, a_1, a_2, a_3, \ldots)$.

d) $L : Z \to Z$ (where Z is as in part (c)) sending $(a_1, a_2, a_3, \ldots) \mapsto (a_2, a_3, a_4, \ldots)$.

e) $I : W \to \mathbb{R}$ (where W is the vector space of continuous real-valued functions on the closed interval $[0, 1]$) sending $f \mapsto \int_0^1 f(x)dx$.

f) $D : V \to V$ (where V is the vector space of infinitely differentiable functions on $\mathbb{R}$) sending $f \mapsto f'$.

g) $E : V \to \mathbb{R}$ (where V is as in part (f)) sending $f \mapsto f(0)$.

h) $Q : V \to V$ (where V is as in part (f)) sending $f \mapsto f^2$.

i) $S : V \to V$ (where V is as in part (f)) sending $f(x) \mapsto f(x)\sin(x)$.

4. For each of the maps in #3 that is a linear transformation, find the following:

a) The *range* of the map, also known as the *image*. (The range of a linear transformation $T : V \to W$ is by definition $\{w \in W \mid w = T(v) \text{ for some } v \in V\}$.)

b) The *nullspace* of the map, also known as the *kernel*. (The nullspace of a linear transformation $T : V \to W$ is by definition $\{v \in V \mid T(v) = 0\}$.)

5. Let V, W, Z be vector spaces. Let $T : V \to W$, $S : V \to W$, and $U : W \to Z$ be linear transformations. Let c be a scalar.

a) Consider the map $T + S : V \to W$ given by sending $v \mapsto T(v) + S(v)$. Is this a linear transformation?

b) Consider the map $cT : V \to W$ given by sending $v \mapsto cT(v)$. Is this a linear transformation?

c) Consider the map $U \circ T : V \to Z$ by given sending $v \mapsto U(T(v))$. Is this a linear transformation?