Math 370 Problem Set #3 Due week of Feb. 3, 2014, in lab.

Read Hoffman and Kunze, Chapter 2, Section 3.

1. From Hoffman and Kunze, Chapter 2, do these problems:
Pages 39-40, #2,5-7. Pages 48-49, #1-5.

2. Prove that the functions e^x, e^{2x}, e^{3x} are linearly independent in the real vector space V consisting of differentiable functions. (Hint: If not, then differentiate twice.)

3. a) Find all real numbers α such that the vectors $(\alpha, 1, 0)$, $(1, \alpha, 1)$, $(0, 1, \alpha)$ are linearly independent in $\mathbb{R}^3$.

b) Does your answer change if instead you work over the rational vector space $\mathbb{Q}^3$ or over the complex vector space $\mathbb{C}^3$ (and allow α to be in $\mathbb{Q}$ or $\mathbb{C}$ respectively)?

4. a) Express $u = (7, -1, 5)$ as a linear combination of $v = (3, -1, 2)$ and $w = (1, 1, 1)$.

b) Show that $(0, 0, 1)$ is *not* a linear combination of v and w.

c) Can $(0, 0, 1)$ be expressed as a linear combination of the vectors u, v, w? (Hint: You don't need to do any computations for this part, once you've done the previous two parts.)

5. Let V be the set of solutions to the differential equation $f''(x) - 3f'(x) + 2f(x) = 0$ and let W be the set of solutions to the differential equation $f'(x) = f(x)$.

a) Show that V and W are real vector spaces, and that W is a subspace of V. (In particular, you have to show that W is contained in V.)

b) Find a basis for W, and the dimension of W.

c) Extend your basis of W to a basis of V (i.e. find a basis of V that contains your basis of W), and find the dimension of V.