Math 350 Sample Final Exam December 2005

1. Show that there are infinitely many primes of the form $6k - 1$, with k a positive integer.

2. Convert each of these decimal numbers to base 3:
   a) $(31)_{10}$
   b) $(.1111....)_{10}$

3. Find the g.c.d. and l.c.m. of 231 and 154.

4. Find all solutions, mod 158, to the congruence $68x \equiv 2 \pmod{158}$.

5. Find the least non-negative residue of 16^{12001} mod 13.

6. Find the inverse of 35 mod 52, or else show that there is no such inverse.

7. Find all solutions to the system of simultaneous congruences

$$x \equiv 1 \pmod 2, \ x \equiv 2 \pmod 3, \ x \equiv 3 \pmod 5.$$

8. Find the order of 3 mod 23. Is 3 a primitive root mod 23?

9. Find all roots of the congruence $x^2 \equiv 14 \pmod{125}$.

10. Find the values of $\phi(42), \sigma(42), \tau(42), \mu(42)$.

11. Suppose that $f(n)$ is an arithmetic function with the property that for all positive integers n, we have that $\sum_{d|n} f(d) = [\sqrt{d}]$. (Here the sum is over the positive divisors, and $[x]$ means the greatest integer in x.) Find the value of $f(20)$.

12. Explain what a digital signature is, what its purpose is, and how it works.

13. Determine if 2 is a square mod 57.

14. Determine whether 25 is a fourth power mod 37.

15. a) Find the continued fraction expansion for 27/16.
    b) Do the same for $1 + \sqrt{3}$.