Math 314 Sample Exam #3 April, 2019

Instructions: This is a sample exam for Exam #3 in Math 314 (held on Wednesday, May 1). Like the actual exam, it consists of five problems. Do all five, showing your work and *explaining your assertions*. Each problem is worth 10 points, for a total of 50 points. Partial credit is given as appropriate on the exam.

Note: Extra credit will be given to those who hand in this sample exam in lecture on Mon., April 29, or who submit it to their TA by 8:30pm on Tuesday, April 30 (e.g. at the Tuesday lab, which all are invited to).

1. Let $A = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$. Find the characteristic and minimal polynomials of A over $\mathbb{R}$. Is A diagonalizable over $\mathbb{R}$? If it is, find a diagonal matrix that is similar to A. Justify your assertions.

2. Show that if a 2×2 real matrix A has the property that $A^2 = -I$, then A cannot be diagonalized over $\mathbb{R}$ but that it can be diagonalized over $\mathbb{C}$.

3. Let $V = \mathrm{M}_3(\mathbb{R})$ be the vector space of real 3×3 matrices. Define $P : V \to V$ by $P(A) = \frac{1}{2}(A - A^{\mathrm{t}})$.

a) Show that P is a linear transformation.

b) Show that P is a *projection*, i.e. $P^2 = P$.

c) Determine the kernel and image of P, and find their dimensions.

4. For $v = (a, b)$ and $w = (c, d)$ in $\mathbb{R}^2$, define $\langle v, w \rangle = ac - ad - bc + 2bd$. Show that this is an inner product, and find an orthonormal basis of $\mathbb{R}^2$ with respect to this inner product.

5. Find real numbers a, b such that with respect to the usual inner product (dot product) on $\mathbb{R}^3$, there is an orthonormal basis of $\mathbb{R}^3$ consisting of eigenvectors of $\begin{pmatrix} 1 & 0 & a \\ 0 & 2 & b \\ -1 & 3 & 0 \end{pmatrix}$.

Explain.