Math 314 Sample Exam #2 March, 2019.

Instructions: This is a sample exam for Exam #2 in Math 314 (held on Friday, March 29). Like the actual exam, it consists of five problems. Do all five, showing your work and *explaining your assertions*. Each problem is worth 10 points, for a total of 50 points. Partial credit is given as appropriate on the exam.

Note: Extra credit will be given to those who hand in this sample exam to their TA by 3pm on Wednesday, March 27, or who submit it in lecture on March 27.

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by $T(a, b) = (b, a)$.
   a) Find the matrix of T with respect to the standard basis $\{e_1, e_2\}$ of $\mathbb{R}^2$.
   b) Do the same with respect to the basis consisting of $f_1 = (1, 1)$ and $f_2 = (-1, 1)$.

2. Let V be a finite dimensional vector space and let v, w be non-zero vectors in V. Show that the annihilators of v and of w are equal if and only if v and w are linearly dependent.

3. Define $T : \mathbb{R}^3 \to \mathbb{R}^4$ by $T(a, b, c) = (a - b, b - c, c - a, 2a - b - c)$. Find the rank and the nullity of T. Do the same for the transpose linear transformation $T^* : (\mathbb{R}^4)^* \to (\mathbb{R}^3)^*$ on the dual spaces.

4. Prove that the real matrices $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{pmatrix}$ are not similar. [Hint: Use determinants.]

5. Let A be a 3×3 matrix, and suppose that $A^5 = 0$. Show that the minimal polynomial of A is of the form x^n for some positive integer $n \leq 5$. Show by example that A is not necessarily the zero matrix.