Note: This problem set is due on Tuesday, April 30. It can be submitted either in lecture on Monday, April 29, or to the TA by the end of lab on Tuesday, April 30. All students are invited to attend the Tuesday labs, since there will be no Thursday lab and since Exam #3 will be in class on Wednesday.

Reminder: Exam #3, given in class on Wednesday, May 1, will cover the material since Exam #2; i.e., the material starting with PS 10. Sample Exam 3 is available on the course web page. Those who submit the sample exam in lecture on April 29, or who submit it to their TA by the end of lab on Tuesday, April 30, will get extra credit. There are also more study problems for Exam 3 on the course web page (as an aid to study, not for additional extra credit). During the actual exam, no electronic devices, notes, or other aids can be used, except for a single two-sided handwritten 5"x7" index card.

Read Hoffman and Kunze, Chapter 8, Sections 2-7.

1. From Hoffman and Kunze, Chapter 8, do these problems: pages 288-290, #3,4,9; pages 298-299, #1,4; pages 308-311, #1,8; pages 317-318, #1,9.

2. Let V be an inner vector space, and let W be a subspace of V.
   a) Show that $W \subset (W^\perp)^\perp$.
   b) Show that $W = (W^\perp)^\perp$ if V is finite dimensional. [Hint: If $\dim V = n$ and $\dim W = d$, then what is $\dim W^\perp$? $\dim (W^\perp)^\perp$?]

3. In $\mathbb{R}^4$, let W be the span of $(1, 0, 1, 0)$ and $(1, 1, 3, 1)$.
   a) Find an orthonormal basis of W. [Hint: Gram-Schmidt.]
   b) Find the point on W closest to $(1, 2, 3, 4)$.
   c) Express $(1, 2, 3, 4) = w_1 + w_2$ explicitly, where $w_1 \in W$ and $w_2 \in W^\perp$.
   d) Find an orthonormal basis of $W^\perp$.

4. Use Gram-Schmidt to find an orthonormal basis for the subspace of $\mathbb{C}^3$ spanned by $(1, -1, i)$ and $(i, 1, i)$, with respect to the usual Hermitian inner product.

5. Over $\mathbb{R}$, which of the following matrices have an orthonormal basis of eigenvectors? What about over $\mathbb{C}$? Also, which of these matrices are symmetric? orthogonal? Hermitian? unitary?

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$