Math 314 Problem Set #12 Due week of April 15, 2019, in lab.

Read Hoffman and Kunze, Chapter 6, Sections 5-7.

1. From Hoffman and Kunze, Chapter 6, do these problems: Pages 205-206, #1-3,6,8; page 208, #1(a),2; page 213, #1,4; pages 218-219, #2,3.

2. Let $A \in M_n(\mathbb{R})$. Suppose that A is triangularizable over $\mathbb{R}$, and that it is diagonalizable over $\mathbb{C}$. Prove that A is diagonalizable over $\mathbb{R}$.

3. Suppose that $A \in M_5(\mathbb{R})$ has characteristic polynomial equal to $(x-1)(x^2+2)(x-3)^2$.
a) Can A be diagonalizable over $\mathbb{R}$?
b) If A is diagonalizable over $\mathbb{C}$, find the minimal polynomial of A.
c) If A is not diagonalizable over $\mathbb{C}$, find the minimal polynomial of A.
Explain.

4. Find the smallest field containing $\mathbb{Q}$ with the property that the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

is triangularizable. Determine whether A is diagonalizable over that field.

5. Determine which of the following matrices are diagonalizable over $\mathbb{R}$. For each of them, do this in two ways: first by using the criterion in terms of the characteristic polynomial and the dimensions of the eigenspaces, and second by using the criterion in terms of the minimal polynomial.

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ 0 & 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}.$$