Math 314 Problem Set #4 Due week of Feb. 11, 2019, in lab.

In Hoffman and Kunze, read Chapter 2, Sections 5-6, and Chapter 3, Section 1.

1. From Hoffman and Kunze, do these problems:
   a) In Chapter 2, pages 48-49, #7; page 55, #2; page 66, #3.
   b) In Chapter 3, pages 73-74, #1,5,8,9,10.

2. Which of the following are linear transformations?
   a) $T : \mathbb{R}^3 \to \mathbb{R}^4$ sending $(x, y, z) \mapsto (x - y, y - z, z - x, 0)$.
   b) $M : \mathbb{R}^2 \to \mathbb{R}$ sending $(x, y) \mapsto xy$. (Here we view $\mathbb{R} = \mathbb{R}^1$.)
   c) $R : Z \to Z$ (where Z is the vector space of sequences of real numbers) sending $(a_1, a_2, a_3, \ldots) \mapsto (0, a_1, a_2, a_3, \ldots)$.
   d) $L : Z \to Z$ (where Z is as in part (c)) sending $(a_1, a_2, a_3, \ldots) \mapsto (a_2, a_3, a_4, \ldots)$.
   e) $I : W \to \mathbb{R}$ (where W is the vector space of continuous real-valued functions on the closed interval $[0, 1]$) sending $f \mapsto \int_0^1 f(x)dx$.
   f) $D : V \to V$ (where V is the vector space of infinitely differentiable functions on $\mathbb{R}$) sending $f \mapsto f'$.
   g) $E : V \to \mathbb{R}$ (where V is as in part (f)) sending $f \mapsto f(0)$.
   h) $Q : V \to V$ (where V is as in part (f)) sending $f \mapsto f^2$.
   i) $S : V \to V$ (where V is as in part (f)) sending $f(x) \mapsto f(x)\sin(x)$.

3. For each of the maps in #2 that is a linear transformation, find the following:
   a) The *range* of the map, also known as the *image*. (The range of a linear transformation $T : V \to W$ is by definition $\{w \in W \mid w = T(v) \text{ for some } v \in V\}$.)
   b) The *nullspace* of the map, also known as the *kernel*. (The nullspace of a linear transformation $T : V \to W$ is by definition $\{v \in V \mid T(v) = 0\}$.)

4. Let $S : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation taking $(x, y) \in \mathbb{R}^2$ to $(a, b, c) \in \mathbb{R}^3$ whenever

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}.$$

Find the kernel and image of S, describing them both geometrically and in terms of equations.

5. Let V be a real vector space, and suppose that $S : V \to \mathbb{R}$ and $T : V \to \mathbb{R}$ are linear transformations. Define $P : V \to \mathbb{R}^2$ by $P(v) = (S(v), T(v))$.
   a) Show that P is a linear transformation.
   b) Find the kernel of P in terms of the kernels of S and T.