Math 314 Problem Set #1 Due week of Jan. 21, 2019, in lab.

Read Hoffman and Kunze, Chapter 1, and Section 1 of Chapter 2. (The material in Chapter 1 after Section 1 is a review of material from Math 240.)

1. a) Is $\mathbb{R}$ a vector space over the field $\mathbb{Q}$?

b) Is $\mathbb{Q}$ a vector space over the field $\mathbb{R}$?

c) Is the set of purely imaginary complex numbers a vector space over $\mathbb{R}$? (A complex number $a + bi$ is called "purely imaginary" if $a = 0$. Here b is real.)

d) Is the set of complex numbers of absolute value 1 a vector space over the field $\mathbb{R}$?

e) Is the set of symmetric 3×3 real matrices (i.e. matrices with $a_{ij} = a_{ji}$ for all i, j) a vector space over $\mathbb{R}$?

f) Is the set of invertible 3×3 real matrices a vector space over $\mathbb{R}$? (A matrix A is called "invertible" if there is a matrix B such that AB and BA are both the identity matrix.)

2. Which of the following sets is a field? For those that are, why? For those that are not, why not?

a) the set of 2×2 real matrices (under matrix addition and multiplication).

b) the set of irrational real numbers (under addition and multiplication of real numbers).

c) the set of complex numbers of the form $a+bi$ with a, b each rational (under addition and multiplication of complex numbers).

d) the set $\mathbb{R}^2$ under the usual addition of vectors, and with multiplication defined by $(a, b) \cdot (c, d) = (ac - bd, ad + bc)$. (Hint: You've seen this set before, but it's in disguise here.)

3. Let c be a fixed real number. Let $f_0(x), f_1(x), f_2(x)$ be real-valued differentiable functions. Let V_c be the set of all 3-times differentiable functions $y(x)$ that are solutions to the homogeneous linear differentiable equation

$$y''' + f_2(x)y'' + f_1(x)y' + f_0(x)y = c.$$

Show that V_c is a vector space over the field of scalars $\mathbb{R}$ if and only if $c = 0$.

4. a) Write down the addition and multiplication tables of a field having exactly three elements.

b) Do the same for a finite field of some other number of elements, greater than three and less than ten. (Caution: You want to be sure that what you write down is a field. For example, every element other than 0 must have a multiplicative inverse.)