From Apostol, read Chapter 1, sections 16, 17, 19-21, 24, 27. Read Chapter 3, section 4 (beginning with Theorem 3.4). Read Chapter 5, sections 1-6. Read Chapter 12, sections 1-6. Optional: Read Chapter 1, sections 6, 18, 22, 23, 25; and Chapter 3, sections 18, 19.

1. From Apostol, 1.26, page 83: do problems 21(a), 22(a), 23.

2. From Apostol, 5.5, pages 208-209: do problems 4, 12, 14, 26.

3. From Apostol, 12.8, pages 456-457: do problems 1(a,c,d), 3, 4, 20.

4. For each of the following functions f on the closed interval $[0, 10]$, determine whether there is a continuous function F on $[0, 10]$ such that F is differentiable on the open interval $(0, 10)$ and $F'(x) = f(x)$ for all x in $(0, 10)$.
   a) $f(x) = [x]$.
   b) $f(x) = \sin(\sin(x))$.

5. Let α be a real number, and consider the sequence $a_1, a_2, a_3, \ldots$, where $a_i = i\alpha - [i\alpha]$ (fractional part of $i\alpha$) for all $i \geq 1$.
   a) Show that if $\alpha \in \mathbb{Q}$, then for every $i \geq 1$ there is an $n \geq 1$ such that if $a_j \neq a_i$ then $|a_i - a_j| \geq \frac{1}{n}$.
   b) Show that if $\alpha \notin \mathbb{Q}$, then for every $i \geq 1$ there is *no* integer $n \geq 1$ having the above property. That is, for every $i \geq 1$ and for every $n \geq 1$, show that there exist terms $a_j \neq a_i$ such that $|a_i - a_j| < \frac{1}{n}$. [Hint: By partitioning $[0, 1]$ into small subintervals of length $1/n$ and considering which terms lie in each subinterval, show that for each n there is *some* i such that $0 < |a_i - a_j| < \frac{1}{n}$ for some $j > i$. Then show for each n that the same is the case for *every* i.]

6. Let V be the set of continuous functions on the closed interval $[0, 1]$. We may add two functions by defining $(f + g)(x)$ to be $f(x) + g(x)$, and we may multiply a function by a real number by defining $(cf)(x) = c(f(x))$.
   a) Show that the set V, under these two operations, satisfies all the vector laws (given in and just after Theorem 12.1, page 447 of Apostol, and also listed as axioms 1-10 in Section 15.2, pages 551-552).
   b) Show that if we define $f \cdot g = \int_0^1 f(x)g(x)dx \in \mathbb{R}$, then the laws of dot product (listed in Theorem 12.2 of Apostol, page 451) are also satisfied.