Math 104, Lecture 2 Sample Exam 3 March 25, 2011

Instructions: This sample exam consists of ten questions. Do them all, giving yourself 50 minutes. For each problem, indicate the correct answer, and also present your work, showing how you arrived at your answer. While working on this exam, you may refer to a two-sided hand-written sheet of notes, not to exceed $8\frac{1}{2}$ by $5\frac{1}{2}$ inches. No other notes should be consulted, and you should not use any electronic devices such as calculators or computers. If you submit this complete exam to your TA by 1pm on Thursday, March 31, showing your answers and your work, you will receive extra credit.

1. Consider the region in the plane where $x^2+y^2 \leq 1$, and $x, y \geq 0$. What is the y-coordinate of the centroid of this region?

(a) $3/4\pi$ (b) $4/3\pi$ (c) $3\pi/4$ (d) $4\pi/3$ (e) $12/\pi$ (f) $\pi/12$

2. If a fair coin is flipped four times, what is the probability that there will be exactly two heads?

(a) 0 (b) 1/8 (c) 1/4 (d) 3/8 (e) 1/2 (f) 1

3. Let $f(x) = 4 - x^2$ for $-2 \leq x \leq 2$, and let $f(x) = 0$ for all other values of x. If k is a real number, then $kf(x)$ is a probability density function

(a) for all real values of x. (b) for no real value of x.
(c) for all $x \geq 0$ but not for any $x < 0$. (d) just for $k = 5/17$.
(e) just for $k = 3/32$. (f) just for $k = \pi$.

4. The *sequence* $a_n = \dfrac{2n+1}{3n+2}$

(a) converges to 0. (b) converges to 1/2. (c) converges to 2/3.
(d) converges to 3/5. (e) converges to $\ln(2)$. (f) diverges.

5. The *series* $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{2^{2n}}$

(a) converges to $-3/7$. (b) converges to $-4/7$. (c) converges to $-3/4$.
(d) converges to 5/3. (e) converges to 9/4. (f) diverges.

6. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{(n^2+n)^{2s}}$. Which of the following describes the set of real numbers s for which this series converges?

(a) $s < 1$ (b) $s > 1$ (c) $s < 1/2$ (d) $s > 1/2$ (e) $s = 0$ (f) no values of s

7. The series $\displaystyle\sum_{n=1}^{\infty} \frac{1+e^{-n}}{n}$

(a) converges by the comparison test. (b) diverges by the comparison test.
(c) converges by the ratio test. (d) diverges by the ratio test.
(e) converges by the integral test. (f) diverges because the terms do not approach 0.

8. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[4]{n}}$

(a) converges by the absolute convergence test.
(b) diverges by the absolute convergence test.
(c) converges by the alternating series test.
(d) diverges by the alternating series test.
(e) converges by the geometric series test.
(f) diverges by the geometric series test.

9. Which of the following series converges absolutely?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n}$ (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$ (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n n^2$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ (e) $\displaystyle\sum_{n=1}^{\infty} (-1)^n 2^{-n}$ (f) $\displaystyle\sum_{n=1}^{\infty} (-1)^n$

10. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n}$.

(a) 0 (b) 1/2 (c) 3/4 (d) 1 (e) 3/2 (f) 2