Instructions: This sample exam consists of ten questions. Do them all, giving yourself 50 minutes. For each problem, indicate the correct answer, and also present your work, showing how you arrived at your answer. While working on this exam, you may refer to a two-sided hand-written sheet of notes, not to exceed $8\frac{1}{2}$ by $5\frac{1}{2}$ inches. No other notes should be consulted, and you should not use any electronic devices such as calculators or computers. If you submit this complete exam to your TA by 1pm on Friday, Feb. 25, showing your answers and your work, you will receive extra credit.

1. Evaluate $\displaystyle\int_4^6 \frac{4x-6}{x^2-4x+3}\,dx$.

(a) $\ln(45)$ (b) $\ln(6)$ (c) $\cos^{-1}(3)$ (d) $\sin(35)$ (e) $\dfrac{\pi}{6}$ (f) $\sqrt{52}$

2. Find $\displaystyle\int \frac{2x+3}{x^2+1}\,dx$.

(a) $\ln(\dfrac{x^2+x}{x-1})+C$ (b) $\tan^{-1}(\dfrac{x}{x^3+x})+C$ (c) $\sec^{-1}(x+2\sqrt{x})+C$
(d) $\sin^2(1/x)+2x^2+C$ (e) $\ln(x^2+1)+3\tan^{-1}(x)+C$ (f) e^{2x+1}/x^2+C

3. A certain differentiable function has the properties that $\sec(3x)f'(x)=x$ and $f(0)=1$. What is $f(\pi/2)$?

(a) $\dfrac{4}{5}+\dfrac{\pi}{2}$ (b) $\dfrac{4}{3}-\dfrac{\pi}{7}$ (c) $\dfrac{8}{9}-\dfrac{\pi}{6}$ (d) $\dfrac{10}{11}+\dfrac{\pi}{3}$ (e) $\dfrac{7}{6}-\dfrac{\pi}{8}$ (f) $\dfrac{5}{4}+\dfrac{\pi}{5}$

4. Find $\displaystyle\int \tan^3(x)\sec^2(x)\,dx$.

(a) $\sin(2x)+\frac{1}{2}\cos(2x)+C$ (b) $\ln\left|\sec(x)+\tan(x)\right|+C$ (c) $\tan^3(x)+\sec^2(x)+C$
(d) $\dfrac{x^2+x-1}{x^2-x+1}+C$ (e) $\frac{1}{3}\sec^3(x)+C$ (f) $\frac{1}{4}\tan^4(x)+C$

5. Find $\displaystyle\int \frac{x+1}{\sqrt{9-x^2}}\,dx$.

(a) $\tan^{-1}(3x)+\sqrt{9-x^2}+C$ (b) $\cos^{-1}(x+1)/\sqrt{9-x^2}+C$ (c) $\ln(x+1)\cdot\sqrt{9-x^2}+C$
(d) $\sin^{-1}(x/3)-\sqrt{9-x^2}+C$ (e) $(9-x^2)/\sec(x+1)+C$ (f) $\cos^{-1}(\sqrt{9-x^2})+\ln(x/9)+C$

6. Using the table on integrals on the last page of this exam, find $\displaystyle\int \frac{dx}{x\sqrt{1-4x^2}}$.

(a) $\sin^{-1}(\dfrac{x}{2})+C$ (b) $\frac{1}{2}\ln\left|\dfrac{\sqrt{1-4x^2}-1}{x}\right|+C$ (c) $\left|2x+\sqrt{4x^2-1}\right|+C$
(d) $\ln\left|\dfrac{\sqrt{1-4x^2}-1}{2x}\right|+C$ (e) $\sqrt{1-4x^2}+\ln\left|\dfrac{\sqrt{1-4x^2}+1}{x}\right|+C$ (f) $\dfrac{1}{2}\sec^{-1}(\dfrac{x}{2})+C$

7. Which of the following integrals is approximated by the sum

$$\cos^3(1) + 2\cos^3(1.2) + 2\cos^3(1.4) + \cdots + 2\cos^3(2.8) + \cos^3(3)?$$

(a) $\int_1^3 \cos^3(x)\,dx$ (b) $\int_1^3 10\cos^3(x)\,dx$ (c) $\frac{1}{10}\int_1^3 \cos^3(x)\,dx$

(d) $\int_1^3 \sin^3(x)\,dx$ (e) $\int_1^3 10\sin^3(x)\,dx$ (f) $\int_1^3 \frac{1}{10}\sin^3(x)\,dx$

8. The integral $\int_2^\infty \frac{x+4}{x^3}\,dx$

(a) converges to -1. (b) converges to 0. (c) converges to 1.
(d) converges to 2. (e) converges to π. (f) diverges.

9. Which of the following integrals corresponds to the arclength of the graph of $y = x - \tan^{-1}(x)$ from $x = 0$ to $x = 1$?

(a) $\int_0^1 2\pi(x - \tan^{-1}(x))\,dx$ (b) $\int_0^1 \pi(x - \tan^{-1}(x))^2\,dx$ (c) $\int_0^1 \pi x^2 - \pi(\tan^{-1}(x)^2)\,dx$

(d) $\int_0^1 \sqrt{x^2 + \tan^{-1}(x)^2}\,dx$ (e) $\int_0^1 \sqrt{1 + \frac{x^4}{(1+x^2)^2}}\,dx$ (f) $\int_0^1 \sqrt{1 + \sec^{-1}(x)^2}\,dx$

10. Which of the following integrals corresponds to the area of the surface obtained by rotating the graph of $y = \sin(x)$, from $x = \pi/4$ to $x = \pi/2$, around the x-axis.

(a) $2\pi \int_{\pi/4}^{\pi/2} \sin(x)\sqrt{1 + \cos^2(x)}\,dx$ (b) $2\pi \int_{\pi/4}^{\pi/2} \sin^2(x)\sqrt{1 + \cos(x)}\,dx$

(c) $2\pi \int_{\pi/4}^{\pi/2} \cos^2(x)\sqrt{1 - \sin^2(x)}\,dx$ (d) $\int_{\pi/4}^{\pi/2} \sqrt{1 + \cos^2(x)}\,dx$

(e) $\int_{\pi/4}^{\pi/2} \sqrt{1 + \cos(x)}\,dx$ (f) $\int_{\pi/4}^{\pi/2} \sqrt{1 - \cos^2(x)}\,dx$

Short table of integrals

$$\int \sqrt{u^2+c}\;du = \frac{u}{2}\sqrt{u^2+c} + \frac{c}{2}\ln\left|u+\sqrt{u^2+c}\,\right| \tag{1}$$

$$\int \sqrt{b^2-u^2}\;du = \frac{u}{2}\sqrt{b^2-u^2} + \frac{b^2}{2}\sin^{-1}(\frac{u}{b}) \tag{2}$$

$$\int \frac{du}{\sqrt{u^2+c}} = \ln\left|u+\sqrt{u^2+c}\,\right| \tag{3}$$

$$\int \frac{du}{\sqrt{b^2-u^2}} = \sin^{-1}(\frac{u}{b}) \tag{4}$$

$$\int \frac{\sqrt{b^2\pm u^2}}{u}du = \sqrt{b^2\pm u^2} + b\ln|\frac{\sqrt{b^2\pm u^2}-b}{u}| \tag{5}$$

$$\int \frac{\sqrt{u^2-b^2}}{u}du = \sqrt{u^2-b^2} - b\cos^{-1}|\frac{b}{u}| \tag{6}$$

$$\int \frac{du}{u\sqrt{b^2\pm u^2}} = \frac{1}{b}\ln|\frac{\sqrt{b^2\pm u^2}-b}{u}| \tag{7}$$

$$\int \frac{du}{u\sqrt{u^2-b^2}} = \frac{1}{b}\sec^{-1}(\frac{u}{b}) \tag{8}$$