Math 104, Lecture 2 Sample Exam 1 Feb. 1, 2011

Instructions: This sample exam consists of ten questions. Do them all, giving yourself 50 minutes. For each problem, indicate the correct answer, and also present your work, showing how you arrived at your answer. While working on this exam, you may refer to a two-sided hand-written sheet of notes, not to exceed $8\frac{1}{2}$ by $5\frac{1}{2}$ inches. No other notes should be consulted, and you should not use any electronic devices such as calculators or computers. If you submit this complete exam to your TA prior to the actual exam, you will receive extra credit. Consult with your TA for details of when and how to submit it.

1. Which of the following limits does *not* exist? Explain.

(a) $\lim_{x\to 0} \dfrac{x+1}{x-1}$ (b) $\lim_{x\to 1} \dfrac{x^2-1}{x-1}$ (c) $\lim_{x\to 1} \dfrac{x+1}{x-1}$

(d) $\lim_{x\to 0} |x|^3$ (e) $\lim_{x\to 0} \dfrac{\sin(x)}{\cos(x)}$ (f) $\lim_{x\to 1} \dfrac{x-1}{x+1}$

2. Suppose that a certain function f satisfies $f(0) = 0$ and $f(1) = 2$; that $f'(x) > 0$ for all x; and that $f''(x) < 0$ for all x. Which of the following is a possible value of $f(2)$?

(a) -1 (b) 0 (c) 1 (d) 2 (e) 3 (f) 4.

3. Evaluate $\displaystyle\int_1^4 \big(\frac{x-1}{x^2} + 3\sqrt{x}\big)\, dx$.

(a) $2\frac{7}{8} - \frac{\pi}{6}$ (b) $13\frac{1}{4} + \ln 4$ (c) $e^3 - 2^{\frac{1}{4}}$ (d) $\cos(\frac{4}{3}) - \ln 2$ (e) $172\frac{1}{2} - \sqrt{3}$ (f) .001

4. Find the area of the region that lies between the graph of $y = \sin(x)$ and the x-axis, between $x = -\pi$ and $x = \pi$.

(a) 0 (b) 1 (c) 2 (d) -2 (e) 4 (f) -4

5. Find $f'(0)$, where $f(x) = \sin(\pi e^x)$.

(a) -2π (b) $-\pi$ (c) 0 (d) π (e) 2π (f) 3π

6. Consider the region in the plane that is bounded above by the graph of $y = e^x$, below by the line $y = 1$, and on the right by $x = 1$. If the region is rotated about the x-axis, what is the volume of the resulting solid?

(a) 3 (b) π^2 (c) $e^{\frac{3}{2}}$ (d) $\pi + \sqrt{2+e}$ (e) $\cos(2e)$ (f) $(e^2-3)\pi/2$

7. A solid has its base on the x, y-plane, and its height is 2. For each number z with $0 \le z \le 2$, the horizontal cross section of the solid by a plane at height z above the x, y-plane is a square whose sides have length equal to $(z-2)^2$. Find the volume of the solid.

(a) 1/2 (b) 17/3 (c) 32/5 (d) $\pi/7$ (e) $e/9$ (f) $\sqrt{2}/10$

8. Find the average value of the function $\cos(x)\ e^{\sin(x)}$ on the interval $0 \le x \le \pi/2$.

(a) $3/2\pi$ (b) 0 (c) $-2/\pi$ (d) $\frac{2(e-1)}{\pi}$ (e) $\ln(\pi/2)$ (f) $2e\pi$

9. Evaluate $\displaystyle\int_1^e \frac{\ln x}{x^2}\,dx$.

(a) $1 - \dfrac{2}{e}$ (b) $e^2 - e^{-2}$ (c) $2\pi(e^2+1)$ (d) $\dfrac{1}{e^2}$ (e) $\ln(e-1)$ (f) 0

10. Which of the following is an anti-derivative of $\sin^3 x + \dfrac{1}{1+x^2}$?

(a) $\frac{1}{4}\sin^4(x) + \ln(1+x^2)$ (b) $3\sin^2(x)\cos(x) - 2x/(1+x^2)^2$ (c) $x^4 + x^3 - 2x$

(d) $\sin^2(x) + \frac{1}{3}\sec(x) + \frac{\pi}{2}x$ (e) $\sqrt{1-x^2} + \dfrac{e^x - e^{-x}}{2}$ (f) $\arctan(x) + \frac{1}{3}\cos^3(x) - \cos(x)$