Math 104, Lecture 2 Problem Set 13 Spring 2011

Due the week of April 18, 2011, in recitation.

Review section 12.10 of Stewart, and read sections 12.11 and 10.1.

Do the first seven core problems for section 12.10; all the core problems for section 12.11; and the first five core problems for section 10.1; but do not hand them in.

The problems below are to be handed in.

Part A: Do the following problems from Stewart:

Section 12.10: 7, 16, 25, 29, 31, 43, 51, 55.
Chapter 12 true-false quiz (p. 795): 1-10.
Section 10.1: 1, 4, 5.

Part B: Do the following problems:

1. For each of the following functions $f(x)$, determine if there is a Maclaurin series for f. If there is, find it, and determine where it converges to f. [Hint: In some cases, there is a shorter way than using the formula for the terms of the Maclaurin series.]

a) $f(x) = \cos(2x^2)$.
b) $f(x) = (e^x + 1)^2$.
c) $f(x) = x^5 + 4x^3 - 2x + 12$.
d) $f(x) = 1/\sqrt{1-x}$.
e) $f(x) = \arcsin(x)$.
f) $f(x) = \sqrt[3]{x}$.
g) $f(x) = \sin(x)/x$ for $x \neq 0$; $f(0) = 1$.
h) $f(x) = \cos(x)/x$ for $x \neq 0$; $f(0) = 1$.

2. For each of the following, find a function with the given series as its Maclaurin series. For which values of x does it converge to the function?

a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{(n+2)!}$ b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n-1}x^n}{3^{n+1}}$ c) $\displaystyle\sum_{n=0}^{\infty} (n+1)x^n$

d) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+2}}{n+1}$ e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ f) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$ g) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

3. a) Does the function $f(x) = 1/x$ have a Maclaurin series? Does it have a Taylor series at $x = 1$? Explain why or why not; and if such a series does exist then find it.

b) Do the same for the function $\ln(x)$.

c) Find the Maclaurin series for $1/(x+1)$ and $\ln(x+1)$, and compare them to your answers in parts (a) and (b).

d) In general, explain the relationship between the Taylor series for a function $f(x)$ at a value $x = a$ and the Maclaurin series for the function $g(x) = f(x+a)$.

e) Find and compare the Maclaurin series for $\cos(x)$ with the Taylor series for $\sin(x)$ at $x = \pi/2$. Give an explanation for what you observe.

4. a) If k, n are non-negative integers with $k \geq n$, explain why the binomial coefficient $\binom{k}{n}$ is equal to $k!/n!(n-k)!$ and why $\binom{k}{n}$ is equal to $\binom{k}{k-n}$. (It is also equal to the number of ways of choosing n things from a set of k things.)

b) Consider the pattern formed as follows by these binomial coefficients:

$$\begin{matrix} & & & & \binom{0}{0} & & & & \\ & & & \binom{1}{0} & & \binom{1}{1} & & & \\ & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & & \\ & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} & \\ \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} \end{matrix}$$

Continuing this for several more rows, and evaluating each of the binomial coefficients, describe all patterns that you can find among these numbers. (This configuration is called "Pascal's triangle.")

c) Explain the relationship between the numbers that appear in a given row and the probabilities associated to the outcomes of flipping a coin a certain number of times.

5. For each of the following differential equations, find as many functions $f(x)$ as you can that are solutions. [Hint: In each case, consider functions of the form e^{ax}, $\sin(ax)$, $\cos(ax)$, and polynomials.]

a) $f'(x) = 2f(x)$. b) $f''(x) = -4f(x)$.

c) $f''(x) + f'(x) = 6f(x)$. d) $f'(x) = f(x) - x^2$.