Math 104, Lecture 2 Problem Set 12 Spring 2011

Due the week of April 11, 2011, in recitation.

Read Stewart, sections 12.8-12.10.

For sections 12.8 and 12.9, work through the first six core problems, but do not hand them in.

The problems below are to be handed in.

Part A: Do the following problems from Stewart:

Section 12.6: 5, 8, 33, 38, 40.

Section 12.7: 2-12 (even numbers only)

Section 12.8: 4-8, 10-12, 29, 30, 32, 33.

Section 12.9: 3, 4, 6-8, 11.

Section 12.10: 5, 6, 8, 10.

Part B: Do the following problems:

1. Suppose that a series $a_1 + a_2 + a_3 + \cdots$ diverges to infinity, and that a series $b_1 + b_2 + b_3 + \cdots$ converges to some real number.

a) Explain why the series $a_1 + b_1 + a_2 + b_2 + a_3 + b_3 + \cdots$ diverges.

b) Do the same for $a_1 + b_1 + b_2 + a_2 + b_3 + b_4 + a_3 + b_5 + b_6 + \cdots$.

2. a) Consider the series

$$1 + \frac{1}{4} + \frac{1}{7} + \frac{1}{10} + \frac{1}{13} + \frac{1}{16} + \frac{1}{19} + \ldots.$$

Does it converge? diverge to infinity? diverge but not to infinity?

b) Do the same for the series

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{5} - \frac{1}{6} + \frac{1}{8} - \frac{1}{9} + \frac{1}{11} - \frac{1}{12} + \ldots.$$

c) Do the same for the series

$$1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} + \frac{1}{8} - \frac{1}{9} + \frac{1}{10} + \frac{1}{11} - \frac{1}{12} + \frac{1}{13} + \ldots.$$

[Hint: In part (c), use parts (a) and (b) together with problem 1(b).]

d) Does the alternating series test apply in part (c)? Explain.

3. For $n = 1, 2, 3, \ldots$, let $a_n = \left(\frac{n+1}{n}\right)^n$.

a) Find $\lim_{n\to\infty} \ln(a_n)$. [Hint: L'Hôpital's Rule.]

b) Find $\lim_{n\to\infty} a_n$. [Hint: Use part (a).]

c) Using your answer in part (b) and the ratio test, find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^n}{n!} x^n$.

d) Do the same for the power series $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$.

4. Let $f(x) = 1/(1+x)$.

a) Find the Maclaurin series $\sum_{n=0}^{\infty} a_n x^n$ for $f(x)$ and determine its domain of convergence.

b) Does this series converge at $x = 1/2$? That is, if we set $x = 1/2$ in the Maclaurin series, does the resulting series $\sum_{n=0}^{\infty} a_n \left(\frac{1}{2}\right)^n$ converge? If so, find what the resulting series converges to, and compare this to the value of $f(1/2)$.

c) What happens if instead you take $x = 2$?

5. For each of the four series in problem 3 on Problem Set 11, do the following:

a) Find a real number $c$ and a function $f(x)$ such that the series you are considering is obtained by setting $x = c$ in the Maclaurin series for $f(x)$. [Hint: The desired function appears in an example in Sections 12.9-12.10 of Stewart.]

b) Evaluate $f(c)$ and compare this to the value you conjectured in problem 3(c) on Problem Set 11.