Math 104, Lecture 2 Problem Set 11 Spring 2011

Due the week of April 4, 2011, in recitation.

Read Stewart, sections 12.6-12.7.

For each of these sections, work through the first six core problems, but do not hand them in.

The problems below are to be handed in.

Part A: Do the following problems from Stewart:

Section 12.3: 15, 29, 40.

Section 12.4: 39, 43.

Section 12.5: 31, 32, 36. [Hint for problem 36(a): Use induction. In other words, first show this for $n = 1$. Then show that if it's true for some n then it's true for $n + 1$ also. Since it's true for $n = 1$, it's then true for $n = 2$, $n = 3$, etc.]

Section 12.6: 3, 4, 6, 12, 17, 18.

Section 12.7: 3, 5, 11, 13.

Part B: Do the following problems:

1. If s is a real number and $\displaystyle\sum_{n=1}^{\infty} 1/n^s$ converges, write $\zeta(s)$ for the sum.

a) For which real numbers s does this sum exist?

b) Find the limit of the sequence $\zeta(2), \zeta(3), \zeta(4), \ldots$. [Hint: Compare these numbers to integrals.]

c) For $s = 1, 2, 3$, by taking partial sums, estimate $\zeta(2s)$ to s digits beyond the decimal point.

d) Using your answer to part (c), for each of $s = 2, 4, 6$ find an integer n (depending on s) such that $\zeta(s) = \pi^s/n$.

2. Define a sequence $\{r_n\}_{n=1}^{\infty}$ as follows: $r_1 = 1/6$, and

$$r_n = (-1)^{n+1}\frac{n}{(2n+1)!} + \sum_{i=1}^{n-1}(-1)^{i-1}\frac{r_{n-i}}{(2i+1)!} \quad \text{for } n > 1.$$

a) Find the exact values (as fractions) of r_1, r_2, r_3, r_4. Also find the decimal expansions of each to several digits.

b) Based on your answers to (a), make a guess as to whether the sequence $\{r_n\}_{n=1}^{\infty}$ converges; and if it does, what the limit is.

c) Compare your answers in part (a) with your answers to problem 1(d). What do you notice?

d) Using what you observed in part (c), make a guess about the value of $\zeta(8)$. Also make a guess about the values of $\zeta(10)$ and $\zeta(12)$, using that $r_5 = 1/93555$ and $r_6 = 691/638512875$ (which you can verify if you like!).

3. Consider the following series:

i. $\frac{\pi}{2 \cdot 1!} - \frac{\pi^3}{2^3 \cdot 3!} + \frac{\pi^5}{2^5 \cdot 5!} - \frac{\pi^7}{2^7 \cdot 7!} + \cdots$

ii. $1 - \frac{\pi^2}{3^2 \cdot 2!} + \frac{\pi^4}{3^4 \cdot 4!} - \frac{\pi^6}{3^6 \cdot 6!} + \cdots$

iii. $1 - \frac{2}{9} + \frac{3}{9^2} - \frac{4}{9^3} + \frac{5}{9^4} - \cdots$

iv. $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$

a) Determine whether each of these series converges.

b) For each series that converges, compute a partial sum that appears sufficient to give the first several digits of the sum of the series.

c) Based on your observations, make a conjecture about the exact values of the sums of these series.